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Data validation of gravity field and satellite data using correlation and coherence method

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A B S T R A C T

Satellite data is frequently used as an initial study of a research area for its easy to access feature as well as its improving quality. One of the available satellite data is geopotential data. Satellite data is commonly used to be correlated to the topography data. In this research, satellite data is used as the database of validation in a research area. Valid measuring data is highly required, so that the qualified data is obtained for further process. To identify the validity, additional other QC is required than the existing QC which in 1D. The validation method which applied are correlation and coherence method. The distribution of correlation and coherence values show asimilirity or compatibility of field data with satellite data. The correlation method was calculate in 2D and the coherence in 1D. Correlation calculation of field data produces high correlation and coherence value to satellite data as much as 0.7 to 0.95 so that it could be identified that acquisition and data processing have been carried out correctly.

1. Introduction

The qualified field data /measurement data /terrestrial data are something that needs to be fulfilled in a research, so that valid analysis results could be obtained. To obtain qualified data, a correct procedure of the acquisition process is required. The tools must be calibrated and the raw data have to process correctly. In the gravity method, to obtain valid observation gravity data does not easy because of uncorrect data. Uncorrect data frequently occur because of the sensitivity of the tools, errors in measurement procedure, and errors in station position. To obtain qualified data, the process of quality control (qc) over the measurement data must be carried out. Gravity method already had a method of field data qc, but the current field data qc is in the form of 1D graphic observation and there is no value of its compatibility or similirity. One of the methods to improve data quality is by improving the existing qc method by identifying its compatibility value. Therefore, a comparison between field data and the data that is already validated i.e. satellite data is carried out. Satellite data is obtained based on the concept of physics by calculating complex mathematics. Along with the time and the development of technology, satellite data becomes more detailed and valid since it is the result of more complex mathematical calculation and physics formula. The data whichused as an input in mathematics calculation consists of observation data on field /terrestrial data which already identified and the data of the earth model. One of geopotential satellite data is provided by Curtin University (Perth, Australia) and Technical University of Munich (Germany) is GGMplus gravity data. GGMplus (Global Gravity Model Plus) is a combination of GRACE satellite observation data, GOCE, and terrestrial data.

Satellite data is frequently used to help the analysis process of a method. One of the methods that could use satellite data is the gravity method. In general, gravity measurement in terrestrial or on land takes much time and cost. The availability of satellite data is very helpful for the researchers to be able to analyze more completely and also it encourages the researchers to perform initial study or research before executing to do a terrestrial acquisition. Satellite data is also helpful for the education and research world. Researchers have more opportunities to be able to explore their skills in processing the data especially signal analysis by using satellite data. The current satellite data is available for some types of parameters and is freely provided. One of satellite data which provided freely is gravity field data. Besides gravity data there are also elevation data, weather data, global geoid data, etc. Satellite data providers such as BGI, GOCE, ICGEM, etc. Previous researches use satellite data to be analyzed, the structure under the surface of

Lamongan's geothermal field [1-3]. The use of GRACE satellite data to analyze the cause of geoid changes [4-7]. Analysis of geoid and seismic is performed to analyze the earthquake occurring in Tohoku-Oki by using satellite gravity field data [8].

In the process of gravity data acquisition, one of difficulties that frequently occurs is an unreachable area because of the topography condition. Uneven distribution of data would influence the process of advanced analysis just like in the process of modeling. In general, the gravity field would have a regional anomaly pattern that does not change quickly from time to time, so that satellite data is very possible to be used as complementary data to fulfill the data emptiness in a research area. It could be done if the field data must have compatibility or similirity of anomaly pattern with its satellite data. Therefore, a method is required to identify the compatibility value between the field data and satellite data. The high similirity of terrestrial data towards satellite data enables to prove thatsatellite data can be used as terrestrial complimentary data. The similiraty or compatibility value also can be used to identify if there is an error on the acquisition process, where the value of gravity observation field is not compatible with its concept of physics.

The method of field data validation/terrestrial data towards satellite data that could be used to complement the quality control is correlation and coherence methods. The correlation method is more often used in the seismic method. In the gravity method, the correlation method is commonly used to identify the relationship between the gravity and geology of a research area. Previous researches using correlation method by using gravity data such as determining the correlation value between gravity anomaly model 2 dimension to determine the depth to the buried central structure [9], the design of correlation filter to map the source of volcanic sediment [9] determining cross-correlation value between vertical gradient from the gravity data observed and the vertical gradient of the gravity theory since the source of a point mass in North Chinese geothermal area [17]. The correlation coefficient value is used to describe the distribution of equivalent mass in terms of probability [10], identification of under surface geology by correlation of the gravity and magnetic value [11-12]. Correlation analysis is also applied to field gravity data towards topography [13-14]. The use of correlation method to identify the correlation of geo-technique and geophysics method to provide a solution based on the case of landslide evaluation [14]. The correlation method is also used to research the regional tectonic correlation in Venezuela to analyze the gravity data correlation with the velocity model of the P wave to understand the crust structure in Taiwan area [16]. The coherence method is also used more in the seismic method to identify the coherence between 2 log wells. In the gravity method, the coherence method is used to identify the relationship between gravity and topography value of a research area [17-20].

From the aforementioned previous researches, satellite data has not been used as data validating for terrestrial data/field data in quality control process. In this research, the field data test obtained is carried out towards GGMPlus data. GGMplus is geo-potential data that currently has the best accuracy. GGM Plus has short resolution gravity and wave topography of 10 km which its resolution is improved to ±200 m. This data covers all land areas and the area close to the coast on earth with a latitude of ±60º. GGMPlus data consists of gravity disturbance data, gravity accelaration, gravity disturbance, geoid undulation, as well as vertical component deflection [21]. Using correlation and coherence method, terrestrial towards GGMPlus satellite data, the terrestrial data produced could be more valid and the satellite data could be used as the complementary data in a research area.

2. Theory

Correlation

In general, in the science of statistics, correlation is defined as the level of relationship between two random variables. The correlation between two data set is how far they resemble one another. However, correlation does not always show a causal relationship, and even high correlation is highly likely caused by coincidence. Mathematically, a correlation is stated by correlation coefficient with the value of -1 (never occur at the same time), to 0 (truly independent), to 1 (always occur at the same time).

In physics, signal correlation is a mathematics process that could produce the value of resemblance level between two compared signals. The signals could be meant as a unit of physics which vary in time, distance, room temperature, pressure, or other free variables. Mathematically, a signal could be stated as a function of one or more free variables. The continuing signal is the value of the entire signal value from the value up to infinite. A discrete signal is a value set from some value possibilities. The process of signal analysis could be done by comparing 2 signals to identify the relationship and the compatibility occurring between the signals. The correlation method consists of 2 types i.e. autocorrelation and cross-correlation. If 2 signals are compared with themselves or compared to similar signals, it is called autocorrelation, while if the signal is compared to another signal, it is called cross-correlation. The correlation process effectively highlights the resemblance and reduces irrelevant forms. The application of correlation principles in geophysics is widely varied. Some of them are the correlation used in filtering, deconvolution, and signal characterization. To identify how big the correlation of 2 is, it could be performed by using 1 dimension or 2 dimensions signal correlation. If both signals are contour pattern 2 dimension, then the correlation performed in correlation 2 dimension [22-23].

The correlation that could be used in various applications of digital signal processing is very similar to the convolution method. As in convolution, correlation uses 2 (two) signals to produce the third signal. This third signal is called cross-correlation from two signals or correlation from two signals. If it is known *x[n]* and *y[n]* are two different signals, then cross-correlation and correlation between *x[n]* and *y[n]*are defined as follows:

$$r_{xy}[d] = \lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} x[n]\ y[n+d] \qquad (1)$$

If a signal is correlated with itself or *x[n]* and *y[n]* are equal, then it is called autocorrelation. Autocorrelation is a measure of similarity/compatibility with itself [22]. The equation of correlation for autocorrelation is:

$$r_{xx}[d] = \lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} x[n]\ x[n+d] \qquad (2)$$

Cross-correlation could be executed in various dimensions, one of which is in the form of 1 dimension. To finish cross-correlation for two-dimensional contour data, then mathematically, the cross-correlation normalization mathematics equation of two signals as follows [22]:

$$r_d = \frac{\sum_i [(x[n]-\bar{x})(y[n-d]-\bar{y})]}{\sqrt{\sum_i (x[n]-\bar{x})^2}\sqrt{\sum_i (y[n-d]-\bar{y})^2}} \qquad (3)$$

With *r* as the linear relationship between variable *x* towards *y* and *d* is the shifting. If these variables move together, where both are identical, then the value of *r* = +1. If other variables do not move together, then *r* = 0. If one of the variables moves together but in reverse, then *r* = -1. If *r* is bigger then zero, it is a positive correlation. If *r* is less than zero, it has a negative correlation [22].

Signal correlation would give a correlation value of -1 to 1, if the signal is strongly correlated, it would correlate 1, if it is not correlated then the correlation value is 0. A very strong correlation if the correlation value is 0.7 to 1, strong correlation value has a value of 0.5 to 0.74, moderate correlation if it is correlated 0.25 to 0.5, and weak correlation if the value is 0 to 0.25 [23]. If it is strongly correlated, then the change occurred on parameter *x[i]* would be followed by the change of parameter *y[i].* If *x[i]* increases then *y[i]* would also increase, but what could also happen is that if *x[i]* increases then *y[i]* decreases so that negative correlation value is obtained and the result would be the opposite.

Coherence

There are many approaches in the coherence method, one of which is the signal analysis method. In signal processing, coherence is a statistic that can be used to examine the relation between two signals or data sets. It is commonly used to estimate the power transfer between the input and output of a linear system. If the signals are stationary, and the system function is linear, it can be used to estimate the causality between the input and output. If the signals are non-stationary, the concept of coherence has been extended by using the concept of time-frequency distributions to represent the time-varying spectral variations of non-stationary signals.

The coherence calculation process could be performed easily using the Matlab program [24-25]. Using the commands provided in Mathlab:

Cxy = mscohere(x,y)

Mscohere is used to estimate the magnitude squared coherence estimate Cxy of the input signals x and y using Welch's averaged, modified periodogram method. The magnitude squared coherence estimate is a function of frequency with values between 0 and 1 that indicates how well x corresponds to y at each frequency. The magnitude squared coherence is a function of the power spectral densities (Pxx(f) and Pyy(f)) of x and y and the cross power spectral density (Pxy(f)) of x and y (Matlab), as the following formula [25]

$$C_{xy}(f) = \frac{\left|P_{xy}f\right|^2}{(P_{xx}f)(P_{yy}f)} \qquad (4)$$

x and y must be the same length. For real x and y, mscohere returns a one-sided coherence estimate and for complex x or y, it returns a two-sided estimate[26-27].

Two signals are considered coherence if both signals share the same frequency and amplitude and they also have constant phase difference [24]. If the coherence method is used to identify the coherence between gravity and magnetic signal, then the coherence equation becomes:

$$\gamma_{hb} = \frac{|P_{hb}|^2}{(P_{hh})(P_{bb})} \qquad (5)$$

which h represents topography and b the gravity anomaly. Coherence varies from 0 to 1. If γhb=0, there is no information on gravity from the observed topography. If γhb=1, local isostasy is active [24].

Coherence and correlation have similar principles i.e. looking for compatibility between 2 signals. What differentiates the two methods is that if the signal input is the same signal (e.g. 2 inputs of gravity fields), then the correlation method could be chosen, while if the 2 inputs are different signals (e.g. input of gravity field and topography) cross-correlation method or coherence method could be applied. The use of coherence in the geophysics field is still in 1 dimension. It takes mathematics method development to obtain 2 and 3 dimension coherence. In an advanced development, the correlation method could be developed into correlation 3 dimension.

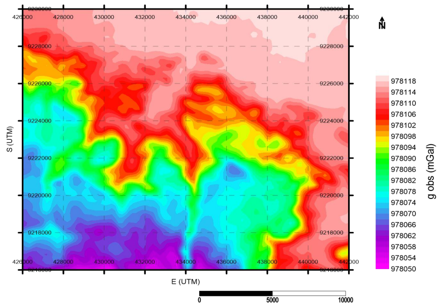
3. Experimental method

To identify the correlation and cohencecoefisient, a signal correlation could be used in 1 and 2 dimensional. Since both are contour patterns, then the analysis performed is 2-dimensional analysis. The correlation method consists of auto-correlation and cross-correlation. Autocorrelation is used if the data correlated are the same type, while for different types of data cross-correlation method is used. In 1dimention could be used commonly statistical method. This reseach used coherence method in 1 dimention. The data which be used are gravity data and geoid data. The data was download from GGMPlus. The coordinat system must be tranform in the same coordinat system, geographic or UTM. Both of data shoud have same grid too. In 1 D process must be chose the same line.

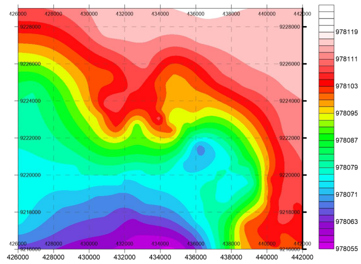
4. Results and discussion

The response of beneath surface of gravity consists of local and regional response. In general, the regional response is caused by big and deep sources. Changes occurring in regional response do not much vary unlike local response, however, since it is a big mass/sources, the response tends to be dominant. Satellite data focuses more on regional response. The regional response of satellite data is obtained from the mathematics calculation process by considering the shape of the earth, the surface of the earth model, the earth's morphology, and the average density of the earth which then is formulated into a mathematical function. Such a mathematical function is used to calculate the value of the gravity field in topography. If the acquisition process on the field is not quite accurate, then the regional response from the field data and satellite data would not have a similar or even the same pattern.

In this research, the correlation process between 2 data: field data and satellite data with a similar type of data in this research used is gravity field data. GGMPlus satellite data has a grid space of 220 m while in-field data, the distance among the stations varied from 500 m to 2000 m and are irregular. The qcmethods which used are the correlation and coherence method.



(a)



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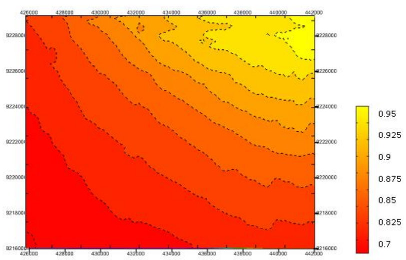


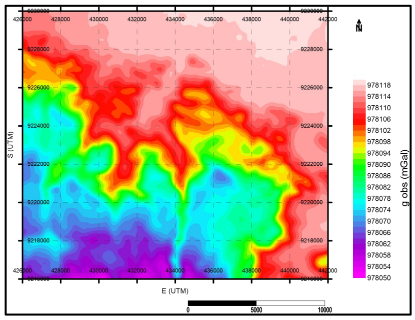
(c)

Fig. 1: (a) Data GGMplus, (b) Data terrestris, and (c) 2D correlation result.

GGMPlus satellite data of a research area for example is Semarang area correlated with field data of Semarang area consisting of 73 stasions. Correlation is performed in 1 dimension/ and 2 dimension. The calculation process of the 2D correlation method applies the convolution process, while the 1D correlation uses the common statistics calculation process. The result is as follows: 1 dimension correlation which is a process of graphic analysis and its compatibility value is calculated using the statistics method. The statistic calculation result produces a compatibility value of 0.93, while the 2 dimension correlation calculation result produces the correlation value of 0.86 to 0.95 (Fig. 1(c)). The correlation valueis categorized into a strong category. Such a result could be analyzed as a compatibility/ similarity response of gravity value in the field with satellitedata. The different values of 1D and 2D correlation are caused by different calculation process, but the result is still in the same range of value. The compatibility of 2 data as the response of forms of the acquisition process in the field that has been done correctly.

In the situation of data acquisition on the field is uncorrect, then the correlation value between the 2 similar data is as much as 0.4 to 0.6 (Fig. 2). In this case, the 1D and 2D correlation values would provide almost similar values 0.3 to 0.6. Such values are included in weak to moderate catagory [14]. As it has been known that it is considered a moderate to strong correlation value if the value is above 0.5. The low correlation value of a measurement area could be information to check the acquisition process that has been carried out.

As known, the correlation value was good if the score was above 0.5. Such worse value became information to check the acquisition process that had been carried out. Since the 2-dimensional correlation compared 2 contour patterns, then the result would be low coeficient of correlation if the contour pattern was different.



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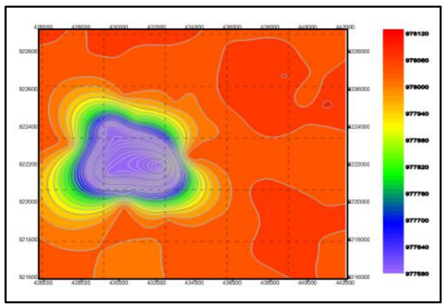

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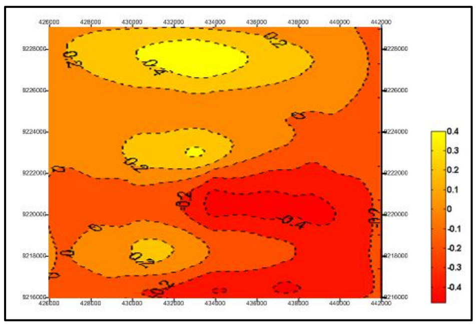


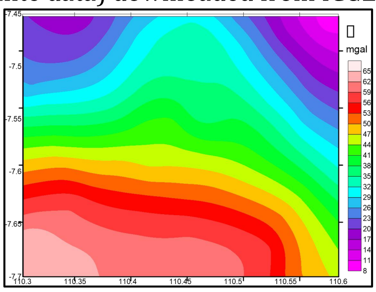
(c)

Fig. 2: (a) GGMplus Data, (b) Terrestrial Data, and (c) 2D Correlation Result.

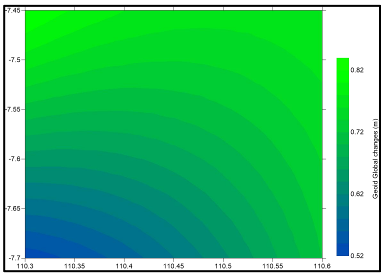
The coefficient of correlation could have a negative to positive value depends on the relationship between the two data. On the example, it did not have a negative value as the data used was the same (Fig. 1(c)). On Fig. 2(c). coeficient of correlation varies from negative to positif. Altought on the same area, both of data set have different contour pattern. Different pattern cused different respon. Negative coefficient commonly discribe some relation between two data like in gravity force, as gravity force with the distance between two mass.

Besides using a similar type of data, it could also be performed analysis on 2 different data, i.e. gravity field data with geoid data or gravity field data with elevation data. If the data used is gravity and topography, negative to positive value would be obtained which shows a contrary relationship i.e. the higher the topography the gravity field value would be smaller and vice versa.The method that can be used for analyze 2 different data type is the cross-correlation method. As in the analysis of the existing correlation value between gravity and magnetic anomaly, it produces a relatively negative correlation value towards anomaly of a magnetic field [10-11]. Data correlation of gravity and topography in spectral-domain could be used to check the elastic thickness and the depth of anomaly structure measured in a causative way, positive correlation value is obtained [13-16]. The correlation value obtained in research could be categorized into a weak, medium, and strong correlation. A strong correlation is when the value close to 0.5 0r -0.5, while a strong correlation is when the value close to 1 or -1 [14].

The relationship between two different data could be calculate using the cross-correlation and coherence method. Fig. 3 is the result of field data cross-correlation (gravity field) towards global geoid data (satellite data) downloaded from ICGEM.



(a)



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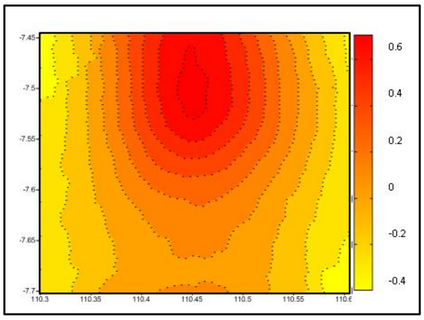

(c)

Fig. 3: (a) TerrestrialData (Bouguer anomaly changes in mGal), (b) Global Geoid Data (ICGEM) (global geoid changes in meter), and (c) 2D Correlation Result.

The result showed in figure 3 shows that there is a positive to negative valued correlation. As explained, a negative value would represent the contrary relationship. The 1D and 2D correlation result provide a similar result, which is -0.4 to 0.7. The correlation result obtained did not show the result expected in the study of gravity and geoid. The existence of such varied values requires deeper study in terms of concept for example. In the case of geoid - gravity field is understood that when gravity field increase its value, the geoid value would also increase. The correlation result shows that not all increasing values of the gravity field are followed by the increasing value of geoid comparably. This is shown by the small value of correlation. If in the concept of physics, there is no equation showing the contrast relationship between geoid - gravity field, then the correlation result must be compared to other methods such as coherence. The result of the coherence method from the 2 different data as in Fig. 4.

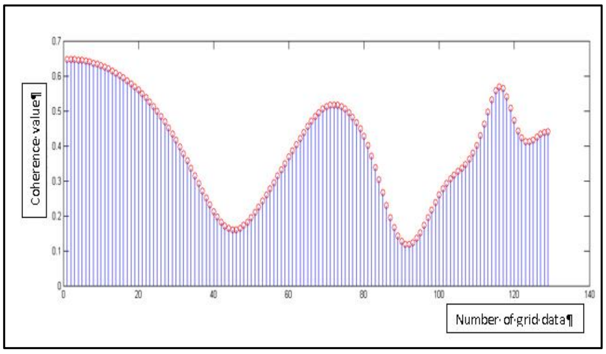

Fig. 4: 1D Coherence Result

1 dimension coherence value obtained provides the value of 0.2 to 0.8. From such distribution value, it could be identified how big is the relationship between 2 signals to provide positive value. The relationship between 2 different data could also be identified clearly by using coherence. Because of the coherence graphic in 1D, so we have to analyze some cross section. We can choose the cross section which have low value of cross-corelation. In this case low value of coherence cused by different respon of gravity and geoid. Gravity paremeter in mGal and geoid in meter. Gravity data is a respon of local and regional mass, but the geoid is a respon of mass as a regional mass only. Geoid is a height of equipotensial surface from ellipsoid refference and gravity is a value of gravity field on topography. Actualy equipotensial and gravity field have closely relatioship. The changes of amount of mass will be change the equipotensial mass , so the gravity field will be change too. The incresing value of equipotensial not always as many as increasing value of gravity field. In the coherence graphic discribe when the geoid and gravity have the same respon, the coefficient will be higher [4, 7, 18, 24, 27].

From the result, it could be identified that the current satellite data could be used to help the process of field data validation and could also be used to fill the data in a difficult area by considering the geology aspect and the existing gravity contour pattern in the research area. The gravity contour pattern of both data must have a similar pattern trend. Since satellite data have a sufficiently big short wave of 10 km, then to obtain the very local description is not easy. To analyze two different data, what can be done is using the cross-correlation and coherence method. If the type of two data are same then the method of correlation is sufficient.

5. Conclusions

Valid field data is required in the research process. To obtain quality data, field data validation is required with other validated data for example satellite data. GGMPlus satellite data is the highest resolution data that currently could be used as one of the data sources that could be used to validate field data and to add gravity field data in a research area. Field data compatibility to satellite data must be validated using correlation or coherence data. For the data that are similar, the correlation method could be used but if the data is different, the cross-correlation or coherence method should be used. Field data correlation calculation that could be used as an example produces a high value of correlation and coherence with satellite data which is 0.7 to 0.95 so that it could provide information that the acquisition process and data processing have been done properly.

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