

Comparison Between Seismic Inversion and Seismic Inversion with Bayesian Inference in Acoustic Impedance

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ARTICLE INFO

Article history:

Received: 30 January 2025

Accepted: 16 July 2025

Available online: 31 August 2025

Keywords:

Acoustic Impedance

Reflection Coefficient

Model Based Inversion

Bayesian MCMC

ABSTRACT

Finding reflection coefficient of seismic trace data is very important to be analyzed in some geological features. Reflection coefficient describes the medium of the subsurface based on Acoustic Impedance (AI) data. Model based seismic inversion is one way that can be used to find reflection coefficient of trace seismic. It needs several steps, like generating calculated trace seismic due to the original one before inversion. Unfortunately, the process is very complicated to reach a best result indicated by error value tends to be zero. While Bayesian MCMC offers the easier way, by setting mean and standard deviation values, it will generate calculated seismic trace data automatically with high similarity to the original one. In other words, Bayesian MCMC helping the inversion process to be shorter. Finally, we have proven that Bayesian MCMC gives the better result of reflection coefficient of model based seismic inversion method.

1. Introduction

Acoustic Impedance (AI) is an important thing in seismic methods. It describes the physical properties of subsurface including density and velocity. Furthermore, Acoustic Impedance generates into Reflection Coefficient and convolves with mother wavelet (Ricker) resulting synthetic seismogram. Then the arrangement of some synthetic seismograms forming seismic section can be analyzed for some needs like knowing the structure and guess the reservoir characterization using inversion method, while additional information like physical properties data (porosity and permeability) are very useful for piercing interpretation [1]. In fact, a seismic section that consists of many seismic traces can be assumed as big data, so it is not effective to analyze it one by one. A geophysicist often takes several parts from a seismic section as the interesting area then do the inverse method to get the AI profile there.

Inversion method is a common way in geophysics to predict the subsurface based on some parameter. This method is available in every geophysical method like seismic, gravity, geo-electricity, tomography, etc. In seismic, model-based inversion is becoming a popular method. It works trace by trace in inverse calculation then comparing it to the original trace [2]. If the inverse result is very similar to the original one, we can say that the inversion result already good enough. However, it is quite difficult and needs some trial and error process.

There is an alternative way to solve a geophysical problem is by using machine learning application like Bayesian Markov Chain Monte Carlo (MCMC). It uses a statistical approach in solving the problem including an inverse method, especially for differentiating calculated and observed data. In this paper we use open source AI data from SEG Wiki [3] to be inverted by seismic model based method and Bayesian MCMC and explain the Bayesian MCMC's rule in this case. Totally, there are four AI data that will be convolved with Ricker wavelet to build the seismic trace.

2. Methods

Seismic model based method is included into post stack seismic inversion. This method starts from convolution between reflection coefficient and Ricker wavelet resulting seismic trace. Equation (1) showing the formula of trace seismic (tr)

$$\text{tr} = \text{wavelet} * \text{reflection coefficient} + \text{noise} \quad (1)$$

In reality, noise can be coming from several factors like heterogeneous medium, man or industry activity, effect of gravity etc [4]. Seismic trace in equation (1) in time units as the data while reflection coefficient is the model parameter that we seek. After reflection coefficient is observed, it is important to understand the relationship between reflection coefficient itself and AI like Equation (2)

$$\text{reflection coefficient} = \frac{AI_{i+1} - AI_i}{AI_{i+1} + AI_i} \quad (2)$$

while AI is a product between density and velocity (vp) multiplication like in Equation (3)

$$AI = \text{dens} * vp \quad (3)$$

subscript i in Equation (2) represents the i-th layer of the medium [2,5,6].

Starting the inverse method, it is important to make Equation (2) simpler like Equation (4)

$$d = Gm \quad (4)$$

where d is the trace seismic, G and m are Ricker wavelet as Kernel matrix and reflection coefficient. Kernel Matrix is built based on Equation (1) and we call it as forward modelling equation. Then based on least square theory, the solution of Equation (4) can be seen in Equation (5)

$$m = G^T (GG^T + I\alpha)^{-1} d \quad (5)$$

with I and α represent identity matrix and damping factor [1,7,8].

Equation (5) works trace by trace in a seismic section. Mathematically, in inversion, the user can do it for all traces that he/she wants, but the data will become bigger. It is just depended on the computer's power and capability. Fig. 1 shows the example of model-based inversion result.

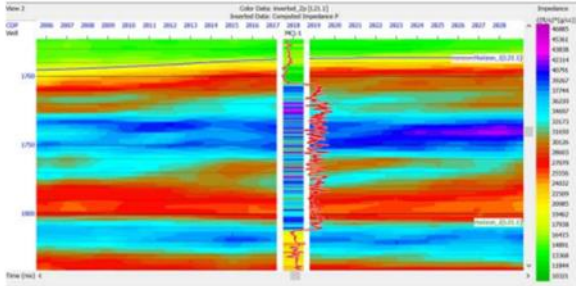


Fig. 1: Example of model based seismic inversion result [9]

Regarding to Fig. 1, the subsurface is represented in the white box in a seismic section. Since the color is aligned to the seismic section profile, the inversion result is good enough [9]. Furthermore, this profile describes the facies as the geological model by some analysis [10,11].

Unfortunately, observing reflection coefficient by model-based inversion method is quite hard. We should care about the little error value between original and inverted seismic trace. An easy way to do that is by differentiating the original seismic trace to calculated seismic trace before doing the inversion by adding some random value to the original trace seismic. The other way, we can produce our calculated seismic trace by using Bayesian MCMC. It forms calculated seismic trace based on some statistical distribution with certain mean and standard deviation values and also the likelihood process. Bayesian MCMC is defined in Equation (6).

$$p(m|d) = \frac{p(d|m)p(m)}{p(d)} \quad (6)$$

with $p(m|d)$ is probability distribution of m to d, $p(d|m)$ is the likelihood function, while $p(m)$ and $p(d)$ are statistical distribution of m and d [7,12]. In Bayesian MCMC, calculated seismic trace is generated by Markov Chain based on mean and standard deviation given by the user. Markov Chain tries to make calculated seismic trace with high

similarity to the original one, then it will be multiplied to the likelihood function. Finally, Monte Carlo as the statistical approach will determine the high similarity and selected one distribution as the result [13–15]. Next, we will see that reflection coefficient by using Bayesian MCMC gives the better result. All of this explanation is represented in flowchart diagram like in Fig. 2.

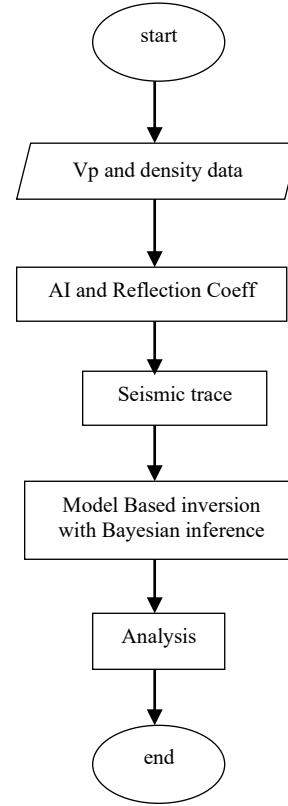


Fig. 2: Flowchart of processing data

However, Bayesian MCMC can be assumed as a part of stochastic inversion. It tries to seek the best model or posterior model from the prior model, in this case is seismic trace. An advantage of Bayesian MCMC is the flexibility, that we can use any parameter but as geophysicist we should care about the parameter itself. Like a research done by [16] that using Bayesian MCMC for make a posterior model for seismic trace and wavelet extraction. Both of prior and posterior model are generated by statistical distribution and we can say that Bayesian MCMC is a good method for sampling the parameter by using statistical approach [17] while likelihood in Equation (6) act like least square method in Equation (5) [18].

3. Result and Discussion

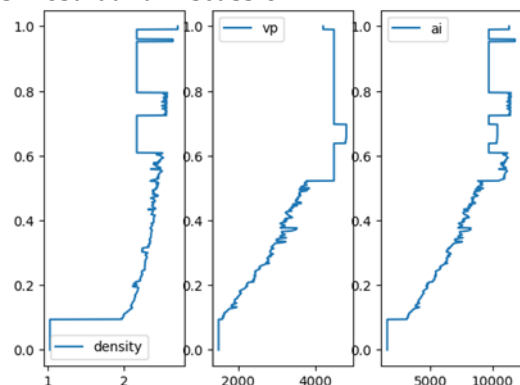


Fig. 3: Density, velocity (vp) and AI of the second well [3]

In this research, we use open source data of density and velocity log from [3] to calculate AI. Actually, there are totally four wells with the similar pattern, and one of them we use in processing data, that is coming from 2nd well that shown in Fig. 3, while for seismic section example, we can see in Fig. 4. To get trace seismic, it is important to convolve reflection coefficient and Ricker wavelet (Fig. 5).

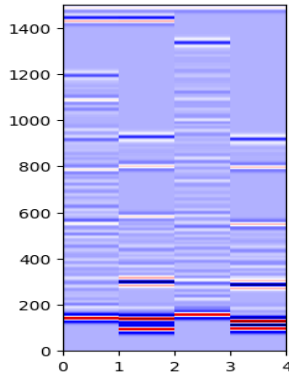
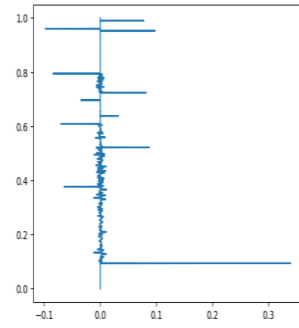
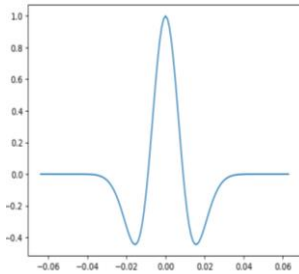


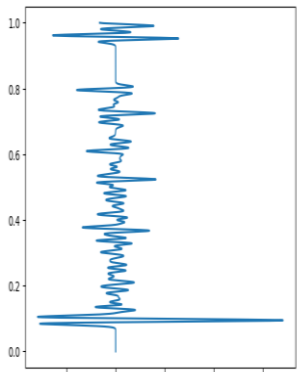
Fig. 4: Seismic section of four seismic traces [3]



(a)



(b)



(c)

Fig. 5: (a) reflection coefficient of second well; (b) Ricker wavelet; (c) trace seismic of second well

Starting the model-based inversion by generating calculated seismic trace in two ways. The first way, we add the random value to original trace seismic and the second way by using normal distribution in Bayesian MCMC with mean and standard deviation

values are 0 and 0.1. For the Kernel Matrix, we arrange from the Ricker wavelet like in Fig. 6.

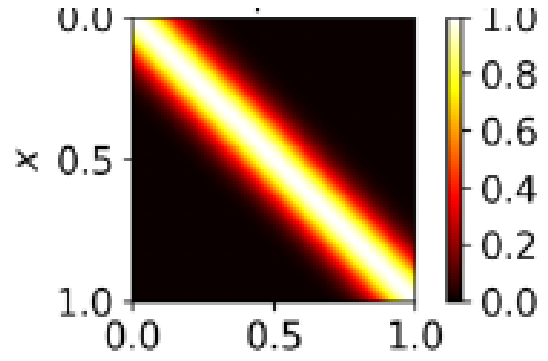


Fig. 6: Illustration of kernel matrix in model based inversion [19]

Then we do model-based inversion, and the result can be seen in Fig. 7.

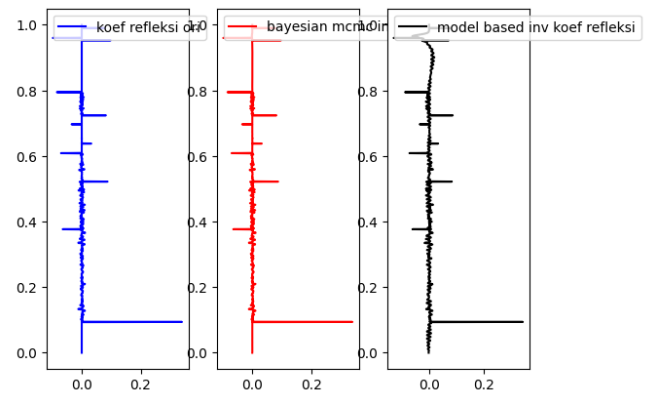


Fig. 7: Blue, red and black represent original reflection coefficient, Bayesian MCMC reflection coefficient and inverted reflection coefficient by adding random value in original trace seismic of second well.

Based on Figure 7, it can be seen that Bayesian MCMC reflection coefficient has high similarity with original one with error value is about 0.0013 the other one is about 0.0022. To make sure that this is the best model, we should refer to the sample trace of Bayesian MCMC itself (Figure 8). The sample trace describes the level of chaotic process in generating calculated seismic trace. In Bayesian MCMC, more chaotic the sample trace means the better result.

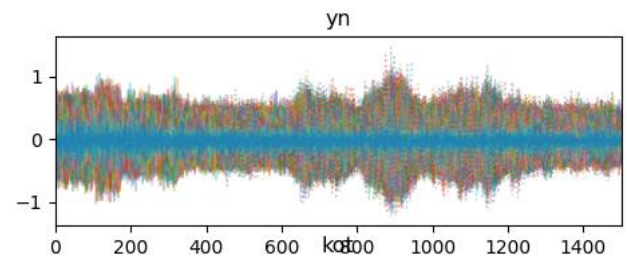


Fig. 8: Sample trace of Bayesian MCMC calculation

There are 1501 iterations in Figure 8. It is similar to the number of seismic trace data. This is the important thing in Bayesian MCMC, we should set the iteration same to the number of data. Anyway, this rule just prevails for continue data like seismic trace. It is very different with the single input data that we can use any iteration number. Nevertheless, generally Bayesian MCMC is reliable to be used in

model based seismic inversion with the best result and easy way.

4. Conclusion

Model based inversion is a common way in seismic to get the reflection coefficient from trace seismic data. One important thing to do this inversion is finding calculated seismic trace that has high similarity to the original one. Unfortunately, this is a quite hard way, we need to add some random value to the original seismic trace until we find the best one. Anyway, this step can be simpler by using Bayesian MCMC algorithm. Setting the standard deviation to generates calculated trace seismic, Bayesian MCMC provides many choices of calculated trace seismic data and choose the best one. In this paper, it has been proven that, reflection coefficient of Bayesian MCMC gives the better result than the other.

Acknowledgment

This Research supported by Basic Research grant No. 121/UN62.21/DT.07.00/2024 LPPM UPN Veteran Yogyakarta.

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