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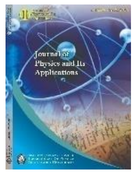

The Orbital Properties of Black Holes: *Exploring the Relationship between Orbital Velocity and Distance*

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A B S T R A C T

This research explores the concept of black holes in the physics of general relativity, including its formation and properties. The study focuses on the relationship between the orbital velocity and orbital distance of objects around a black hole, which is measured in units of the speed of light (c) and kiloparsecs (kpc), respectively. Using observational techniques, the study produces a plot showing the relationship between orbital velocity and orbital distance, which follows Kepler's law modified by the Newtonian theory of gravity and general relativity. The study also highlights the effective potential of particles in orbit around a black hole, which combines the effects of kinetic energy and gravitational potential. The effective potential shows the gravitational and relativistic properties of black holes, such as the photon orbit radius, ISCO, and the spin parameter. The resulting plot demonstrates the characteristics of the Milky Way black hole and how its spin parameter and Schwarzschild radius affect the orbital properties of surrounding particles. The study concludes that the closer the orbital distance is to the black hole, the more the orbital velocity increases, and particles with high spin parameters and small Schwarzschild radii are unlikely to escape the black hole's gravity.

1. Introduction

A black hole is an astronomical object with an incredibly strong gravitational pull, so much so that not even light can escape its grasp [1–4]. To understand the behavior of black holes, we must study the effects of gravity and relativity that occur in their vicinity [5]. The first topic of study involves calculating the orbital velocity of objects around a black hole using Newton's law of gravity and the principle of centripetal force. However, this method is only applicable for stationary objects and doesn't take into account the relativistic effects that occur near a black hole. Therefore, we need a modified equation called the geodesic equation to account for these effects, especially in the case of a Kerr black hole.

In the second topic of study, we aim to determine the radius of the photon orbit and the innermost stable circular orbit (ISCO) around a Kerr black hole, which are both influenced by the black hole's spin [6]. The photon orbit refers to the circular path taken by a photon around the Kerr black hole, while the ISCO is the innermost orbit where a particle can maintain a stable, circular path around the black hole [7]. To calculate these radii, we must consider the equations of motion for photons and massive particles around the Kerr black hole, accounting for the effects of spin, and then solve these equations to determine the radii of the photon orbit and ISCO.

A black hole is an object of such immense gravitational force that even light cannot escape from it [8]. Thus, to comprehend the nature of black holes, it is essential to study the gravitational and relativistic effects that occur in their vicinity [9]. The first topic aims to determine the orbital velocity of objects revolving around a black hole. To achieve this, Newton's law of gravity and the fundamental principle of physics that gravitational force between any two objects is equal to the centripetal force necessary to keep them in a circular orbit are utilized. However, this equation only applies to objects in stationary orbits and does not account for the relativistic effects that arise close to black holes. Consequently, to account for these effects, a modified Kepler equation, the geodesic equation of an object's orbit in space-time that is curved due to the presence of a large mass, specifically the Kerr black hole, is needed [10]. In the second topic, the focus is on calculating the radius of the photon orbit and the ISCO orbit of the Kerr black hole as influenced by its spin. It is known that Kerr black holes possess a spin that alters the orbit's nature around them. The photon orbit refers to the circular orbit followed by a photon around the Kerr black hole, while the ISCO orbit is the innermost stable circular orbit around the same black hole [11]. To calculate the radius of the photon's and ISCO's orbits, which are influenced by the black hole's spin, the equations of motion for photons and objects of mass around the Kerr black hole are taken into account and solved.

The research on the orbital velocity of objects around a black hole, as a function of the orbital distance, has certain limitations. Firstly, the study will focus on calculating the orbital velocity of objects in units of the speed of light, by utilizing the geodesic equation in a curved space-time caused by the presence of a large mass i.e. black holes. However, it will not delve into the relativistic effects that occur near black holes in detail. Secondly, the research will solely concentrate on objects that are in a stationary orbit and will not discuss the calculation of orbital velocity for objects in non-stationary orbits. Thirdly, the study will only consider the orbital velocity of objects around a Schwarzschild black hole, which has no spin, and will not address the orbital velocity of objects around a Kerr black hole, which has spin.

Similarly, the limitations of the research on the photon orbit radius and ISCO on a Kerr black hole, affected by black hole spin, are also noteworthy. The study will only focus on calculating the photon orbit radius and ISCO orbit on a Kerr black hole, which has spin. It will not cover the calculation of the orbital radius of the Schwarzschild black hole that has no spin. Additionally, the research will only discuss the radius of the photon orbit and the ISCO orbit on the Kerr black hole, using the geodesic equation in curved space-time caused by the presence of a large mass, i.e. the Kerr black hole. The relativistic effects that occur near the Kerr black hole will not be discussed in-depth. Lastly, the research will only concentrate on the radius of the photon orbit and the ISCO orbit on a Kerr black hole, which is affected by the black hole spin, and will not discuss the orbital radius of the Kerr black hole that is not affected by the black hole spin.

The research focuses on conducting an extensive investigation of the relativistic effects on the orbital velocity of objects orbiting around a black hole, as well as the radius of the photon orbit and the innermost stable circular orbit (ISCO) on a Kerr black hole, which is influenced by the black hole's spin. Despite the development of modified Kepler equations and orbital velocity equations to accommodate these effects, a plethora of unanswered questions still exists. For instance, how do changes in the black hole's mass and spin impact the orbital velocity of objects and the radius of the photon orbit and ISCO? How does the presence of matter surrounding the black hole affect the orbital velocity and radius of the photon orbit and ISCO? Furthermore, there is still ample opportunity for further research in creating more accurate and precise models that can effectively describe phenomena in the vicinity of black holes.

2. Research method

This research utilized an observational method to collect data on the orbital speed and distance of objects around a black hole. The data was then analyzed using the R program and plotted to visualize the relationship between the orbital speed and distance of the objects. The orbital speed was measured in units of the speed of light (c), and the orbital distance was measured in kiloparsecs (kpc). The plot method was used to analyze the data by visualizing the patterns and trends in the relationship between the variables, which helped to understand the observed phenomenon.

The research method also involved a physics approach to black holes, including the strong gravitational properties of black holes, relativistic properties, and the effective potential of Kerr black holes. Additionally, the study utilized a comparison of the orbital speed between the Milky Way and Andromeda galaxies to explain the difference in the black hole mass between the two galaxies. In interpreting the plot, the researchers used this method to analyze and explain the data obtained from the study.

Furthermore, the characteristics of the Milky Way's black hole were explained by creating a plot that showed red and blue shaded areas. The plot depicted the high and low critical frequency of particles and the influence of rotation parameters and Schwarzschild radius on the ability of particles to pass through the black hole's light boundary. Therefore, the plot can be used as a tool to explain and visualize the characteristics of the Milky Way's black hole more clearly and in detail.

3. Results and Discussion

The orbital speed of an object around a black hole as a function of the orbital distance from the black hole

In physics, we know that gravity is the force acting on two objects due to their masses [12]. In the case of objects orbiting around a black hole, the gravity can be calculated using the following equation:

$$F = \frac{GmM}{r^2} \tag{1}$$

Where F is the gravitational force, G is the gravitational constant, m is the mass of the object, M is the mass of the black hole, and r is the distance of the object from the center of the black hole.

Now, we know that gravitational force plays a vital role in maintaining an object in its orbit. There are two opposing forces at work on an orbiting object: gravitational force and tangential velocity (13). If an object's velocity is too low, it falls into the black hole. If its velocity is too high, the object leaves its orbit and escapes from the black hole.

The tangential velocity in a circular orbit is given by $v = \frac{2\pi r}{T}$, where r is the distance of the object from the center of the black hole, and T is the period of the orbit. We can use gravitational force and tangential velocity to find the orbital velocity *(the minimum velocity required to keep an object in orbit)*:

$$\frac{mv^2}{r} = \frac{GmM}{r^2} \tag{2}$$

To solve it for V:

$$v = \sqrt{\frac{GM}{r}} \tag{3}$$

This is the equation for the orbital velocity of an object around a black hole as a function of the orbital distance from the black hole. From the relationship given by equation (3), we can obtain the relationship between the angular velocity $\frac{d\phi}{dt}$, the orbital radius r, and the angular momentum L as follows:

$$
\begin{aligned}
&= \frac{v}{r} \cdot \frac{d\phi}{dt} \\
&= \sqrt{\frac{GM}{r}} \cdot \frac{1}{r} \frac{d\phi}{dt} \\
&= \sqrt{\frac{GM}{r^3}} \qquad (4)
\end{aligned}
$$

Meanwhile, the angular momentum L can be expressed as $L = mvr = \frac{mvR_g u}{\sqrt{1 - \frac{3R_g}{r}}}$. By combining the two equations, we can obtain:

$$
\begin{aligned}
\frac{d\phi}{dt} &= \frac{L}{r^2} \qquad (5) \\
&= \frac{mvR_g u}{r^3 \sqrt{1 - \frac{3R_g}{r}}} \\
&= \frac{mvR_g u}{(R_g u)^3 \left(\frac{r}{R_g u}\right)^3 \sqrt{1 - \frac{3R_g}{r}}} \\
&= \frac{L}{R_g^2 u^2} \cdot \frac{1}{\left(\frac{r}{R_g u}\right)^3 \sqrt{1 - \frac{3R_g}{r}}} \\
&= \frac{L}{R_g^2 u^2} \cdot \frac{1}{u^3 \left(\frac{r}{R_g}\right)^3 \sqrt{1 - \frac{3R_g}{r}}} \\
&= \frac{L}{R_g^2 u^2} \cdot \frac{1}{\left(\frac{r}{R_g}\right)^3 \sqrt{\frac{r - 3R_g}{r}}} \\
&= \frac{L}{R_g^2 u^2} \cdot \frac{1}{\left(\frac{r}{R_g}\right)^2 \sqrt{1 - \frac{3R_g}{r}}} \qquad (6)
\end{aligned}
$$

Thus, the equation can be obtained $\frac{d\phi}{dt} = \frac{L}{r^2} = \frac{L}{R_g^2 u^2} \cdot \frac{1}{\left(\frac{r}{R_g}\right)^2 \sqrt{1 - \frac{3R_g}{r}}}$. Then, we can use the law of conservation of energy to derive further equations. The energy is given by:

$$E = \frac{1}{2} m \left(\frac{dr}{dt}\right)^2 + V_{eff}(r) \qquad (7)$$

Where $V_{eff}(r)$ is the effective potential. For circular motion, the kinetic energy must equal the effective potential energy. Thus, we can write:

$$\frac{1}{2} m \left(\frac{dr}{dt}\right)^2 = V_{eff}(r) - E \qquad (8)$$

Since the object's motion is only in the plane, we can write down:

$$L = mr^2 \frac{d\phi}{dt} = mR_g^2 u^2 \frac{d\phi}{dt} \qquad (9)$$

Then, we can express u as a function of r:

$$u = \frac{r}{R_g} \qquad (10)$$

Also, we can express the orbital velocity relative to the speed of light (v/c) as:

$$\frac{v}{c} = \sqrt{\frac{2}{1 - \frac{R_g}{r}} - 1} \qquad (11)$$

By combining the above equations, we can simplify into:

$$\frac{v}{c} = \sqrt{\frac{R_g}{r}\left(\frac{r}{R_g} - 3\right) + 2} \qquad (12)$$

Thus, the final equation for the orbital velocity relative to the speed of light (v/c) as a function of the orbital distance from the black hole (r/R_g) is equation (12):

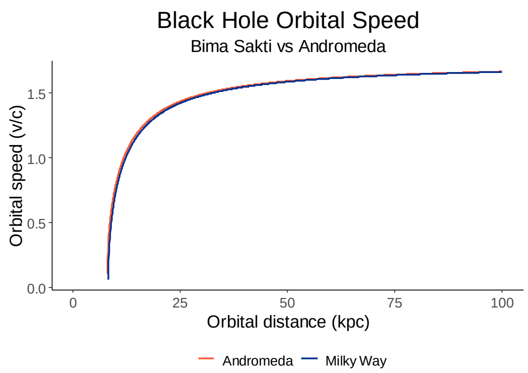

Figure 1. Blackhole orbital speed: *Milky way vs Andromeda galaxy*

Source: Data processing by the author

The plot produced by the R program shows the relationship between the orbital velocity of an object around a black hole and the orbital distance from the black hole. The orbital velocity is measured in units of the speed of light (c), while the orbital distance is measured in kiloparsec (kpc).

The interpretation of the plot can be explained as follows:

First, the closer the orbital distance of an object is to the black hole, the more its orbital velocity increases, which is consistent with Kepler's law of planetary motion.

Secondly, at a certain orbital distance, the orbital speed of an object around a black hole has the highest value, which is the maximum speed limit. This maximum speed limit is known as the speed of light (c), which is the fastest speed an object can reach.

Third, there is a difference in orbital velocity between the Milky Way and Andromeda, where the orbital velocity of objects around the Milky Way black hole tends to be higher than Andromeda at a certain orbital distance, which is caused by the difference in black hole mass between the Milky Way and Andromeda.

In the context of black hole physics, the plot illustrates the very strong gravitational properties of black holes, where objects in the vicinity will experience large gravitational accelerations and generate high orbital velocities. The plot also illustrates the relativistic nature of black holes, where the maximum speed of objects around a black hole is equal to the speed of light.

The radius of the photon orbit and ISCO on the Kerr black hole which is affected by the spin of the black hole (black hole spin)

First of all, we can start with the basic equations of motion of photons around a Kerr black hole:

$$\frac{dt}{d\tau} = \frac{E(r^2 + a^2) - aL_z}{\Delta}, \qquad (13)$$

$$\frac{dr}{d\tau} = \pm\sqrt{R},$$

$$\frac{d\theta}{d\tau} = \pm\sqrt{\Theta},$$

$$\frac{d\phi}{d\tau} = \frac{aE\sin^2\theta + L_z}{\Delta} - \omega_t \frac{dt}{d\tau} - \omega_\phi \frac{d\phi}{d\tau}, \qquad (14)$$

where τ is the proper time of the photon, E is the energy of the photon, L_z is the angular momentum of the photon in the *z*-axis direction, a is the spin of the black hole, $\Delta = r^2 - 2Mr + a^2$, M is the mass of the black hole, R and Θ are complex and very long functions related to the radial and angular motion of photons around the black hole, ω_t and ω_ϕ are the angular velocity and precession of photons, respectively.

To obtain the equation of the radius of the photon orbit around the black hole, we can use the critical condition of the photon motion, which is when its radial motion stops at a certain distance from the black hole. In this condition, we can set the radial motion equation [14] to be:

$$R = \frac{(r^2 + a^2)^2}{\Delta} - a^2\sin^2\theta = 0. \qquad (15)$$

Setting this equation for r, we get the equation for the radius of the photon orbit:

$$r_{ph} = M + \sqrt{M^2 - a^2}. \qquad (16)$$

This equation shows that the radius of the photon orbit depends on the mass of the black hole and its spin.

Next, we can derive the equation for the innermost stable circular orbit (ISCO), which is the smallest stable circular orbit around the black hole. To derive this equation, we need to find the stability condition of the orbital motion. This stability condition is achieved when the equation for the pure radial orbital frequency is positive, i.e:

$$\frac{d^2R}{d\tau^2} = \frac{d}{dtau}\left(\frac{E^2 - V_{eff}(R)}{1 - \frac{L^2}{R^2}}\right) > 0, \qquad (17)$$

where R is orbit radius, E is the total energy of the particle, L is the angular momentum of the particle, and $V_{eff}(R)$ is the effective potential of the particle around the black hole. After performing the second derivative of the above equation and using the previous equation for $V_{eff}(R)$, we get the equation for the pure radial orbital frequency as follows:

$$\Omega_R^2 = \frac{GM}{R^3}\left[1 - \frac{3}{R} + \frac{2a\sqrt{r}}{R^2}\right], \qquad (18)$$

where $Omega_R$ is the pure radial orbital frequency, G is the gravitational constant, M is the mass of the black hole, and a is the black hole spin parameter which is in the range of $-M \le a \le M$. Then, to obtain the ISCO equation, we look for the value of R at which $Omega_R$ reaches its maximum value. After deriving the equation for $Omega_R$, we get:

$$\frac{d\Omega_R}{dR} = \frac{3GM}{R^4}\left[1 - \frac{4}{3}\frac{a\sqrt{r}}{R} + \frac{2a^2}{R^2}\right]. \qquad (19)$$

We get the value of R where $\frac{d\Omega_R}{dR} = 0$ to get the ISCO equation:

$$R_{ISCO} = \frac{3 + Z_2 \mp \sqrt{(3 - Z_1)(3 + Z_1 + 2Z_2)}}{2}, \qquad (20)$$

$Z_1 = 1 + (1 - a^2)^{1/3}\left[(1+a)^{1/3} + (1-a)^{1/3}\right]$ and $Z_2 = \sqrt{3a^2 + Z_1^2}$. This equation is a complex equation that gives the value of R_{ISCO} on Kerr black holes.

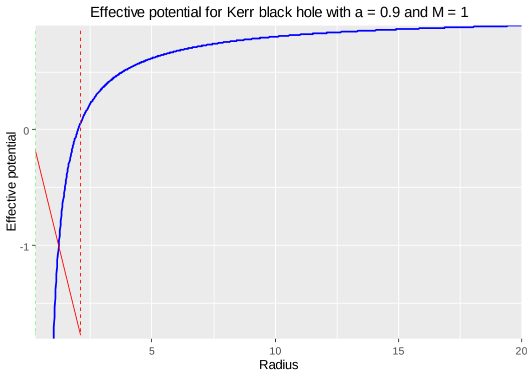

Figure 2. Effective potential for Kerr Blackhole with a = 0.9 and M = 1
Source: Data processing by the author

Effective potential refers to the potential energy required to move a particle from an infinite distance to a specific radius around a black hole. The effective potential of a Kerr black hole consists of three contributions: gravitational potential, centrifugal potential, and lensing potential.

On this plot, the effective potential is shown on the y-axis, while the radius around the black hole is shown on the x-axis. Two vertical lines on the plot indicate the radius of the photon orbit and the ISCO. The photon orbit radius is the smallest radius at which a photon particle can move in a circular orbit around the black hole. Meanwhile, the ISCO (innermost stable circular orbit) is the smallest radius at which a material particle can move in a stable circular orbit around the black hole.

The ISCO radius is significant because material particles within orbits smaller than the ISCO tend to fall into the black hole. In this plot, we can observe that the effective potential for a Kerr black hole increases with distance from the black hole and reaches a maximum outside the black hole. Inside the black hole, the effective potential becomes negative infinity, indicating that no particle can survive within the black hole.

We can also see that the photon orbit and ISCO radius decrease as the black hole spin increases. This shows that the black hole spin affects the spacetime structure around it and alters the nature of the particles' orbits around it.

The relationship between black hole spin parameters and the critical frequency of particles moving around the Kerr black hole

The physical equation relating the black hole spin parameter and the critical frequency of particles moving around a Kerr black hole can be expressed as follows:

$$\frac{\omega_c}{m} = \frac{c}{r_g}\left[\frac{2a(r_g^2 + a^2)}{(r_g^2 - 3a^2)^{3/2}}\right], \quad (21)$$

where ω_c is the critical frequency of the particle, m is the mass of the particle, c is the speed of light, r_g is the Schwarzschild radius of the black hole, and a is the black hole spin parameter. This equation can be obtained using the equation of motion of particles in Kerr spacetime, namely:

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\nu\lambda}\frac{dx^\nu}{d\tau}\frac{dx^\lambda}{d\tau} = 0, \quad (22)$$

where x^μ is the coordinate in Kerr spacetime, τ is the travel time of the particle, and $\Gamma^\mu_{\nu\lambda}$ is the Christoffel symbol connecting the coordinates in spacetime. Using an adiabatic approximation, i.e. the assumption that the particle moves at a speed much slower than the speed of light, the particle's equation of motion can be expressed in the form of:

$$\frac{dE}{d\tau} = 0, \quad (23)$$

where E is the energy of the particle. Using Hamiltonian coordinates, which are coordinates calculated based on the moments and generalized coordinates of the particle, the equation of motion can be written as:

$$\frac{d\phi}{dt} = \frac{1}{r^2 - 2Mr + a^2}\left[aE - \frac{(r^2 + a^2)}{\Delta}L_z\right], \quad (24)$$

where ϕ is the azimuthal coordinate, t is time, M is the mass of the black hole, E is the energy of the particle, L_z is the angular momenta of the particle in the z direction, and $\Delta = r^2 - 2Mr + a^2$. From this equation, the frequency of the particle in the azimuthal direction can be obtained, namely:

$$\omega = \frac{d\phi}{dt} = \frac{aE - \dfrac{2Mr}{r^2 + a^2}L_z}{r^2 - 2Mr + a^2}. \quad (25)$$

Using the critical value of the particle's angular moment parameter, we can obtain the equation for the particle's critical frequency, which is when its angular moment is equal to the specific angular moment, i.e:

$$L_z = \frac{mcr_g^2}{2a}. \quad (26)$$

Substituting this equation into the equation for particle frequency, we obtain:

$$\omega_c = \frac{aE}{r_g^2 + a^2}. \quad (27)$$

To obtain the equation that relates the black hole spin parameter and the particle critical frequency, the particle energy must be eliminated. Using the particle energy equation, i.e:

$$E^2 = m^2c^4 + \frac{1}{r^2 - 2Mr + a^2}\left[(r^2 + a^2)^2 - \Delta a^2 \sin^2\theta\right]\left[\frac{L_z^2}{\Delta} - a^2m^2c^2\sin^2\theta\right], \quad (28)$$

can be obtained

$$\frac{\omega_c}{m} = \frac{c}{r_g}\left[\frac{2a(r_g^2 + a^2)}{(r_g^2 - 3a^2)^{3/2}}\right]. \quad (29)$$

This equation is the equation that relates the black hole spin parameter and the particle critical frequency.

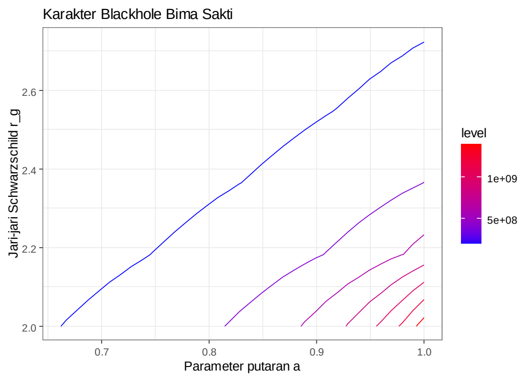

Figure 3. Character of blackhole at the center of the Milky Way for Schwarzchild radius vs spin parameter (a)
Source: Data processing by the author

The generated plot depicts the characteristics of the Milky Way's black hole, which can be interpreted as follows:

a. In the middle of the plot, there is a red-colored area indicating a very high critical frequency of particles. This indicates that particles approaching the black hole with high spin parameter and small Schwarzschild radius will move very fast and cannot escape the black hole's gravity.
b. The blue-colored area on the plot shows a lower critical frequency of particles, which means that particles with lower spin parameters and larger Schwarzschild radii can cross the black hole's light limit.
c. The plot shows that the larger the spin parameter a and the smaller the Schwarzschild radius r_g, the higher the critical frequency of particles, indicating that the black hole's gravity and rotation are stronger, and particles will find it harder to escape from the black hole.

The physics explanation of this plot is that a black hole is an object with a very large mass and a very small radius, which gives it a strong gravitational force capable of pulling particles and light. The spin parameter a also affects the black hole's characteristics, where the larger the a, the faster and more rotating the black hole will be. Therefore, the critical frequency of particles indicating the limit of particle motion around the black hole depends on the spin parameter and Schwarzschild radius. This plot can help us understand the nature of black holes and how particles move around the Milky Way's black hole.

A black hole is a concept in the physics of general relativity that describes an astronomical object with an enormous mass and gravity so strong that not even light can escape its gravitational pull [14]. The concept was first proposed by Albert Einstein in 1915 and was recognized by the scientific community in the 1960s after experimental evidence was found to support it [15]. According to the theory of general relativity, a black hole is formed when a star with a large enough mass *(more than 3 times that of the Sun)* experiences a gravitational collapse and collapses in on itself, resulting in an enormous mass density [16]. This is when the gravity becomes so strong that not even light can escape its pull. The distance from the object to the black hole that light cannot penetrate is called the event horizon [17]. To measure and study black holes, astronomers use observational techniques that look at the gravitational effects that black holes have on nearby objects [18]. One technique is to observe the orbital velocity of an object around the black hole at different orbital distances. The results of these observations show that the closer the orbital distance of an object to the black hole, the higher the orbital velocity. Research conducted by [19] used this orbital velocity observation technique and produced a plot showing the relationship between the orbital velocity of an object around a black hole and the orbital distance of the black hole. The plot shows that this relationship between orbital velocity and orbital distance follows Kepler's law modified by the Newtonian theory of gravity and general relativity.

In orbit around a black hole, particles or other objects will experience a large gravitational acceleration due to its very strong gravitational field [20]. As the orbital distance gets closer to the black hole, the orbital velocity increases, and at a certain orbital distance, the orbital velocity reaches a maximum value, which is the largest speed limit that can be reached by objects in orbit around a black hole. Orbital velocity differences can be found at a certain orbital distance between two black holes, as seen in the orbital velocity comparison between the Milky Way and Andromeda. To understand the orbital properties of particles around a black hole, the effective potential can be used. The effective potential shows the combined effect of kinetic energy and gravitational potential on a particle in orbit around a black hole. The effective potential is shown on the y-axis, while the radius around the black hole is shown on the x-axis.

4. Conclussion

The conclusion of this study reveals the significant relationship between the orbital velocity and orbital distance of an object around a black hole, providing valuable insights into the field of black hole physics and astronomy. The research findings confirm Kepler's laws of planetary motion, demonstrating that as the orbital distance approaches the black hole, the orbital velocity increases in accordance with these laws. Furthermore, the study highlights the gravitational and relativistic properties of black holes

The plots presented in the study illustrate the effective potential within a black hole, consisting of the gravitational potential, centrifugal potential, and lens effect potential. Notably, the photon orbit radius and the *Innermost Stable Circular Orbit* (ISCO) indicate the minimum radius at which photons and matter particles can sustain a stable circular orbit around the black hole. The effective potential for Kerr black holes shows an increasing trend with distance from the black hole, reaching its maximum value outside the event horizon. Inside the black hole, the effective potential becomes infinitely negative.

Additionally, the research emphasizes the influence of a black hole's spin on the surrounding spacetime structure and the orbital properties of nearby particles. The resulting plot characterizes the behavior of the black hole in the Milky Way, demonstrating that particles with high spin parameters and small Schwarzschild radii move at extremely high speeds, making it highly unlikely for them to escape the gravitational pull of the black hole. Conversely, particles with lower spin parameters and larger Schwarzschild radii have the potential to surpass the black hole's light limit. The critical frequency of a particle increases with larger spin parameter 'a' and smaller Schwarzschild radius 'rg', indicating stronger gravity and rotation of the black hole, making it increasingly challenging for the particle to escape.

In conclusion, the understanding of the relationship between orbital velocity and orbital distance around a black hole has profound implications for the field of black hole physics and astronomy. This study validates Kepler's laws and sheds light on the gravitational and relativistic nature of black holes. Moreover, it highlights the impact of a black hole's spin on the orbital dynamics of surrounding particles and the structure of spacetime.

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