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SOME PROPERTIES OF ALMOST JOINTLY PRIME (?? , ??) -SUBMODULES Dian Ariesta Yuwaningsih1*, Burhanudin Arif Nurnugroho2, Dar Mar'atuslia 1, Lolita1 1Department of Mathematics Education, Universitas Ahmad Dahlan, Yogyakarta, Indonesia 2Department of Magister Mathematics Education, Universitas Ahmad Dahlan, Yogyakarta, Indonesia Email: 1*dian.ariesta@pmat.uad.ac.id Abstract. Let ?? and ?? be rings with identity. The definition of prime submodule has been generalized to the almost prime submodule. In addition, the definition of prime submodule has also been carried over to the (?? , ??) -module structure, which is called jointly prime (?? , ??) -submodules.

However, as a generalization of prime submodules, the concept of almost prime submodules has not been carried over to (?? , ??) -module structures. In this paper, we construct the definition of almost jointly prime (?? , ??) - submodules as the generalization of jointly prime (?? , ??) -submodules. We also present several necessary and sufficient conditions for an (?? , ??) -submodule to be an almost jointly prime (?? , ??) -submodule. Keywords: almost prime, jointly almost prime, jointly prime, (?? , ??) -modules I.

INTRODUCTION Every non-zero element is not a zero divisor in an integral domain. If we take any elements and ?? of an integral domain with , we have or . A prime ideal definition emerges in a commutative ring with identity by generalizing the properties of element (or the singleton). Let be a commutative ring with identity and a proper ideal of . The ideal is said to be prime if for each with implies or . As it develops, the prime ideal has been generalized into an almost prime ideal. Following to [1], a non-zero proper ideal of is called an almost prime ideal if for each with implies or .

We already know that the primeness in the ring has been carried over into the module structure. The concept of the prime ideal has been carried over to the module structure into prime submodules. Following [2], a proper submodule of -module is said to be primeif foe every and ?? ? ?? with , implies or . Khashan [3] has generalized a prime submodule into an almost prime submodule in a module over a commutative ring with identity. A proper submodule of -module is said to be almost prime if for each and ?? ? ?? with implies or . We recall that is the annihilator set of .

Furthermore, Beiranvand and Beyrandvand in [4] have defined an almost prime submodule in a module over the non-commutative ring. Following [4], a proper submodule of is said to be almost prime if for every submodule of and ideal ?? of ?? with and implies or . On the other hand, the primeness in the module structure has been carried over to the (?? , ??) - module structure by Khumprapussorn et al. [5]. In [5], Khumprarpussorn et al. have generalized the prime submodule to jointly prime (?? , ??) -submodules.

Furthermore, several other researchers have generlaized the jointlye prime (?? , ??) -submodule, such as left ?? -prime (?? , ??) - submodules in [6], left weakly jointly prime (?? , ??) -submodules in [7], jointly ?? -prime (?? , ??) - submodules in [8], and jointly -prime -submodules in [9]. The concept of almost prime submodules has never been brought to (?? , ??) -module structures. Therefore, through this research, we will develop the almost prime submodule into an (?? , ??) -module structure, which is after this called almost jointly prime (?? , ??) -submodules.

We show that this almost jointly prime (?? , ??) -submodule is a generalization of the jointly prime (?? , ??) -submodule in [5]. Moreover, we also present some properties of the almost jointly prime -submodule. II. THE DEFINITION OF ALMOST JOINTLY PRIME -SUBMODULES Throughout this paper, and ?? are rings with identity and ?? an (?? , ??) -modules, unless otherwise stated. Before we define the almost jointly prime (?? , ??) -submodule, here we give a proposition in [5] which will used in this research.

Proposition 1 [5] Let ?? and ?? be rings, ?? an (?? , ??) -submodule ???? ?? , ?? ? ?? a non-empty set, and ?? ? ?? a non-empty set. If ?? satisfy ?? ? for every , then: 1. If (???? ? ?? , then ?? ? ?? . Moreover, ?? ? (???? . 2. If ????) ? ?? , then ?? ? ?? . Moreover, ?? ? . 3. for every non-empty set of . If is an -submodule of , then . Furthermore, let be an -module and ?? an (?? , ??) -submodule ???? ?? . Then, we define the annihilator of factor (?? , ??) -module ?? / ?? is the set (?? : ?? ??) = { ?? ? ?? | . Following [5], if the ring ?? satisfy ?? 2 = ?? then we can show that the set (?? : ?? ??) form an ideal of ?? .

Below we give the definition of jointly prime (?? , ??) -submodule. Definition 1 [5] Let ?? and ?? be rings and ?? an (?? , ??) -module. A proper (?? , ??) -submodule ?? of ?? is called jointly prime if for each (?? , ??) -submodule ?? of ?? , left ideal ?? of ?? , and right ideal ?? of ?? with ?????? ? ?? implies ?????? ? ?? or ?? ? ?? . Next, here we give another version of the definition of jointly prime (?? , ??)-submodule. Definition 2 [5] Let ?? and ?? be rings and ?? an (?? , ??) -module satisfy ?? = ?? .

A proper (?? , ??) -submodule ?? of ?? is called jointly prime if for each (?? , ??) -submodule ?? of ?? , ideal ?? of ?? , and ideal ?? of ?? with ?????? ? ?? implies ?????? ? ?? or ?? ? ?? . In the following, we define almost jointly prime (?? , ??) -submodules. Definition 3 Let ?? be an (?? , ??) -module and ?? a proper (?? , ??) -submodule of ?? . The submodule ?? is called an almost jointly prime (?? , ??) -submodule of ?? if for every (?? , ??) - submodule ?? of ?? , left ideal ?? of ?? , and right ideal ?? of ?? with ?????? ? ?? and ?????? ? (?? : ?? ??) ???? implies ?? ? ?? or ?????? ? ?? . We can show that the almost jointly prime (?? , ??) -submodule is a generalization of the jointly prime -submodule in the following proposition. Proposition 1 Let ?? be an (?? , ??) -module and ?? a proper (?? , ??) -submodule of ?? .

If ?? is a jointly prime (?? , ??) -submodule of ?? , then ?? is an almost jointly prime (?? , ??) -submodule of ?? . Proof: Let any left ideal ?? of ?? , right ideal ?? of ?? , and (?? , ??) -submodule ?? of ?? with ?????? ? ?? and ?????? ? (?? : ?? ??) ???? . Since ?? is a jointly prime (?? , ??) -submodule of ?? , then from ?????? ? ?? we obtain ?????? ? ?? or ?? ? ?? . Hence, ?? is an almost jointly prime (?? , ??) -submodule of ?? .

? Next, we give several examples of almost jointly prime (?? , ??) -submodules. Example 1 Let Z be an (2Z , 3Z) -module and 6Z an (2Z , 3Z) -submodule of Z . We can show that 6Z is an almost jointly prime (2Z , 3Z) -submodule of Z . Proof: Let any ideal of 2Z , ideal ?? = (3 ??) Z of 3Z , and (2Z , 3Z) -submodule (3 ?? + 1) Z di Z , for an element ?? , ?? , ?? ? Z + . Considering the set (6Z : 2Z Z) = { ?? ? 2Z | ?? Z (3Z) ? 6Z } , so that (6Z : 2Z Z) = (2 ??) Z , for an element ?? ? Z + . Thus, we obtain (6Z : 2Z Z) (6Z) (3Z) = (36 ??) Z 3 .

From this, we obtain 18 ?????? + 6 ????) Z 3 ? 6Z and ?? (3 ?? + 1) Z ?? = (18 ?????? + 6 ????) Z 3 ? (6Z : 2Z Z) (6Z) (3Z) but (3 ?? + 1) Z ? 6Z . Consequently, we get ?? Z ?? = (6 ????) Z 3 ? 6Z . Thus, 6Z is an almost jointly prime (2Z , 3Z) - submodule of Z . ? Example 2 Let Z be an (2Z , 2Z) -module and 4Z an (2Z , 2Z) -submodule of Z . We can show that 4Z is an almost jointly prime (2Z , 2Z) -submodule of Z . Proof: Let any ideal ?? = (2 ??) Z of 2Z , ideal ?? = (2 ??) Z of 2Z , and (2Z , 2Z) -submodule (2 ?? + 1) Z of Z , for an element ?? , ?? , ?? ? Z + .

Considering the set (4Z : 2Z Z) = { ?? ? 2Z | ?? Z (2Z) ? 4Z } , so that (4Z : 2Z Z) = (2 ??) Z , for an element ?? ? Z + . Thus, we obtain (4Z : 2Z Z) (4Z) (2Z) = (16 ??) Z 3 .

From this, we obtain ?? (2 ?? + 1) Z ?? = (8 ?????? + 4 ????) Z 3 ? 4Z and ?? (2 ?? + 1) Z ?? = (8 ?????? + 4 ????) Z 3 ? (4Z : 2Z Z) (4Z) (2Z) but (2 ?? + 1) Z ? 4Z . Consequently, we get ?? Z ?? = (4 ????) Z 3 ? 4Z . Hence, 4Z is an almost jointly prime (2Z , 2Z) - submodule of Z . ? III. SOME PROPERTIES OF ALMOST JOINTLY PRIME -SUBMODULES In this section, we present some properties of the almost jointly prime -submodule.

First, below are given the necessary and sufficient conditions for an -submodule to be an almost jointly prime -submodule. Proposition 1 Let be an -module with and a proper -submodule of . The submodule is an almost jointly prime -submodule if and only if for each ideal of , ideal of , and -submodule of such that and ?????? ? (?? : ?? ??) ???? , implies ?????? ? ?? or ?? ? ?? . Proof: Let any ideal ?? of ?? , ideal ?? of ?? , and (?? , ??) -submodule ?? of ?? such that ?????? ? ?? and ?????? ? (?? : ?? ??) ???? . Since ?? is an almost jointly prime (?? , ??) -submodule, we have ?????? ? ?? or ?? ? ?? .

Conversely, let any left ideal ?? of ?? , right ideal ?? of ?? , and (?? , ??) -submodule ?? of ?? such that ?????? ? ?? and ?????? ? (?? : ?? ??) ???? . It is clear that is an ideal of and is an ideal of . Moreover, we obtain and . Referring to the hypothesis, we obtain or . Thus, or . Hence, is an almost jointly prime -submodule of . Next, we present several necessary and sufficient conditions for an (?? , ??) -submodule to be an almost jointly prime (?? , ??) -submodule. Proposition 2 Let ?? an (?? , ??) -module satisfy ?? ? for each and a proper - submodule of . The following statements are equivalent: 1. is an almost jointly prime -submodule. 2.

For every right ideal of , , and left ideal of such that and ???? ? (?? : ?? ??) ???? , implies ?? ? ?? or ?????? ? ?? . 3. For every right ideal ?? of ?? , (?? , ??) -submodule ?? of ?? , and left ideal ?? of ?? such that ?????? ? ?? and ?????? ? (?? : ?? ??) ???? , implies ?? ? ?? or ?????? ? ?? . 4. For every left ideal ?? of ?? , ?? ? ?? , and right ideal ?? of ?? such that (and (, implies or . 5. For every element , , and such that and , implies or . Proof: (1 ? 2) .

Let any right ideal ?? of ?? , ?? ? ?? , and left ideal ?? of ?? such that ???? ? ?? and ???? ? (?? : ?? ??) ???? . We have (????) = ?? (. Since , we obtain ?? (????) ?? ? ?? . So, we get (????) ? ?? . On the other hand, we have ?? (????) ?? ? ?? (?? : ?? ??) ?????? ? (?? : ?? ??) ???? , so ????) ? (?? : ?? ??) ???? . Since ?? is an almost jointly prime (?? , ??) -submodule, we obtain (????) ? ?? or ?? ? ?? . Thus, we get ?????? ? ?? or ?? ? ?? . (2 ? 3) . Let any right ideal ?? of ?? , (?? , ??) -submodule ?? of ?? , and left ideal ?? of ?? such that ?????? ? ?? and ?????? ? (?? : ?? ??) ???? but ?? ? ?? . Next, let any element ?? ? ?? \ ?? .

We have ?????? ? ?????? ? ?? and ?????? ? (?? : ?? ??) ???? . Referring to the hypothesis, we obtain ?????? ? ?? . (3 ? 4) . Let any left ideal ?? of ?? , ?? ? ?? , and right ideal ?? of ?? such that (and (. It is clear that is an ideal of and is an ideal of . So, we have ????) = (????) ?? (and (. Referring to the hypothesis, we obtain or . Thus, we get or . Let any left ideal of , right ideal of , -submodule of such that and ?????? ? (?? : ?? ??) ???? but ?? ? ?? . Next, let any element ?? ? ?? \ ?? . Since ?? is an (?? , ??) - submodule of ?? , we obtain ?? = ?? so that we get (.

Hence, we obtain and (. Referring to the hypothesis, we get . Thus, is an almost jointly prime -submodule of . Let any element , , and such that and (. Since is a right ideal of , is a left ideal of , then based on the hypothesis, we obtain or . Thus, we get or . Let any right ideal of , -submodule of , and left ideal of such that and ?????? ? (?? : ?? ??) ???? but ?? ? ?? . Next, let any element , we get and . Let any element and ?? ? ?? . We obtain and . Hence, we get and (. Referring to the hypothesis, we obtain so that . Proposition 3 Let ?? an (?? , ??) -module satisfy ?? ? for every and ?? a proper (?? , ??) - submodule of ?? . The following statements are equivalent: 1. ?? is an almost jointly prime (?? , ??) -submodule.

2. For every element ?? ? ?? , (?? , ??) -submodule ?? of ?? , dan element ?? ? ?? with (and (, implies or . 3. For every element , -submodule of , and element with and (, implies or . 4. For every element , -submodule of , and element with and (, implies or . Proof: (1 ? 2) . Let any element ?? ? ?? , (?? , ??) -submodule ?? of ?? , and element ?? ? ?? such that (and (. Since is a submodule of , we get and (. Since is a left ideal of , is a right ideal of ?? , and ?? is an almost jointly prime (?? , ??) -submodule, we have ?? ? ?? or (. Since , we get or . Let any element , -submodule of , and element with and (.

Since is a submodule of , we get so that and (. Referring to the hypothesis, we obtain or . Let any left ideal of , right ideal of , and -submodule of such that and ?????? ? (?? : ?? ??) ???? . Let any element ?? ? ?? dan ?? ? ?? . We obtain , so that and (. Referring to the hypothesis, we get or . Thus, we obtain or . Let any element , -submodule of , and element such that and (. Since and are rings with identity, we get and (. Referring to the hypothesis, we obtain or . Let any element , -submodule of , and element such that and (. Considering that and ?? (. Referring to the hypothesis, we obtain or . IV. CONCLUSIONS AND FUTURE RESEARCH DIRECTION Prime submodules have been generalized to almost prime submodules.

The definition and properties of almost prime submodules have been extended to the structure of -modules to almost jointly prime - submodules. The results of this research will be used as a reference for further research on almost prime submodules in the left multiplication - module and the almost jointly prime radical of -module.
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