JOURNAL OF FUNDAMENTAL MATHEMATICS
AND APPLICATIONS (JFMA) VOL. 8 NO. 2 (2025)
Available online at www.jfma.math.fsm.undip.ac.id

HIERARCHICAL BAYESIAN SMALL AREA ESTIMATION ON OVERDISPERSED DATA: WORKERS WITH DISABILITIES IN INDONESIA

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Abstract. Persons with disabilities encounter difficulties in accessing essential services, including employment, healthcare, information, and political participation. In line with the target 8.5 of the SDGs, efforts have been made to promote full, productive, and decent employment for all, including for persons with disabilities. However, the majority of workers with disabilities in Indonesia remain concentrated in the informal sector during the period of 2022–2023. Unfortunately, data on workers with disabilities is currently only available at the national level. This limitation arises because the sample size of workers with disabilities is insufficient to meet the minimum requirements for direct estimation at the provincial level. Therefore, this research will conduct an applied statistical study using Small Area Estimation approach to assess the participation of persons with disabilities in the workforce at more granular level, such as provinces. In this study, auxiliary variables such as the sex ratio, the number of residents who are shackled, and the availability of computer skills infrastructure were incorporated to the Small Area Estimation (SAE) framework. The Hierarchical Bayesian Poisson-Gamma was employed to improve the precision of direct estimation. The research results show that the HB Poisson-gamma estimator has better precision compared to the direct estimator.
Keywords: Workers with disabilities, SAE, HB Poisson-Gamma

I. INTRODUCTION

Badan Pusat Statistik (BPS) states that employment is an activity of performing tasks with the intention of earning or helping to earn income or profit, for at least 1 hour in the past week (excluding break time) [1]. The 1945 Constitution of the Republic of Indonesia, hereinafter referred to as the 1945 UUD NRI, ensures that every individual has the same right to have access on gainful employment and decent life. Additionally, the 1945 UUD NRI also ensures that every individual will receive fair treatment and just compensation in their work. However, there is still a significant gap in access to employment, especially for persons with disabilities. ILO (International Labor Organization) data shows that 82 percent of persons with disabilities in the world reside in developing countries and live below the poverty line. These individuals with disabilities face difficulties in accessing basic services, such as employment, healthcare, information, and political participation [2].

According to the World Health Organization (WHO), disability is an inherent aspect of the human condition and an integral part of the human experience. This is the result of the interplay between health conditions and/or impairments experienced by an individual, such as dementia, blindness, or spinal cord injuries, and various contextual factors related to environmental factors and different personal behaviors, including societal attitudes, access to infrastructure, discriminatory policies, age, and gender [3]. The term persons with disabilities refers to individuals with impairments, activity limitations, and/or participation restrictions. This should not be confused with disability itself, which pertains to the limitation an individual encounters in functioning. Activity limitation is experienced by person with disability, resulting in difficulties on performing tasks or actions, while participation limitation is also a challenge in engaging in social activities or roles. These obstacles make it difficult for them to interact with their surroundings and hinder active participation in society, including in the workforce. In fact, the inclusion of workers with disabilities is an important part of efforts to achieve sustainable development. The Sustainable Development Goals (SDGs) have set target 8.5, which aims to achieve full and productive employment and decent work for all people, including persons with disabilities, with equal pay for work of equal value by 2030. The approach to measuring economic inclusivity is derived from 8.5.1, which includes the average hourly earnings of male and female workers based on occupation, age, and disability status, as well as indicator 8.5.2, which includes the unemployment rate based on gender, age, and disability status. In addition, SDG goal 10.2 also emphasizes the importance of strengthening social, economic, and political inclusion for all without discrimination, including persons with disabilities.

As a measure of national protection, Indonesia has passed legislation safeguarding the employment rights of persons with disabilities in Law No. 8 of 2016 concerning Persons with Disabilities Article 53. This regulation states that the Government, Regional Government, State-Owned Enterprises (BUMN), and Regional-Owned Enterprises (BUMD) are required to employ at least two percent of persons with disabilities from the total number of employees. Meanwhile, private companies are required to employ at least one percent. This provision serves as an important legal basis in promoting the creation of more inclusive workspaces for persons with disabilities in Indonesia.

Although regulations have been enacted, their implementation is still far from optimal. Data from BPS in Table 1.1 shows the proportion of working residents with disabilities for the years 2022–2023, indicating an increase in the number of workers with disabilities from 720,748 in 2022 to 763,925 in 2023. This increase is also reflected in the proportion of workers with disabilities to the total working population, which rose from 0.53% to 0.55%. Despite this growth, the figures remain very small compared to the total number of workers overall, which reached more than 135 million in 2022 and nearly 140 million in 2023, with the proportion of workers with disabilities still being very low. This indicates that the challenges in accessibility and inclusion for people with disabilities remain significant.

Table 1. Proportion of Working Residents with Disabilities, 2022-2023

	2022	**2023**
Total employed persons with disabilities	720,748	763,925
Total employed population	135,296,713	139,852,377
Proportion of employed persons with disabilities (%)	**0.53**	**0.55**

Source: Badan Pusat Statistik (BPS), Indikator Pekerjaan Layak di Indonesia 2024

Table 2.1 shows that the majority of people with disabilities in Indonesia are still concentrated in the informal sector compared to the formal sector in 2022–2023. Jobs in the informal sector, such as own-account workers (0.85% in 2023), own-account workers assisted by non-permanent workers/family workers/unpaid workers (0.79%), as well as casual workers in agriculture (0.54%) and non-agriculture (0.32%), show a much larger proportion compared to jobs in the formal sector. On the other hand, the proportion of persons with disabilities working as laborers/employees/staff, which is the main indicator of the formal sector, is as low as 0.26% in 2023, although it has slightly increased from 0.23% in 2022. This disparity reflects the low participation of workers with disabilities in the formal sector, which notably offers better job protection and access to social security.

Table 2. Proportion of Working Residents with Disabilities by Employment Status in Indonesia (percent), 2022–2023

Worker Status	2022	2023
Own-account workers	0.81	0.85
Own-account worker assisted by non-permanent workers/family workers/unpaid workers	0.78	0.79
Employer	0.55	0.64
Employee	0.23	0.26
Casual worker in agriculture	0.66	0.54
Casual worker in non-agriculture	0.39	0.32
Unpaid family worker	0.67	0.62

Source: Badan Pusat Statistik (BPS), Indikator Pekerjaan Layak di Indonesia 2024

Unfortunately, data on workers with disabilities is currently only presented at the national level. The number of sample of people with disabilities is small and does not meet the minimum sample size required for direct estimation at the provincial level in Indonesia. Therefore, a Small Area Estimation (SAE) approach is needed to observe the involvement of persons with disabilities in the workforce at a smaller area level, such as provinces or regencies/cities. The SAE method allows for estimation at a small area level, making it easier to identify and address specific and targeted issues.

Several studies on disabilities and workers with disabilities have been conducted, one of which is a study by Maghfirah in 2022 using multinomial logistic regression that examines the employment status of disabled individuals, including unemployment, working in the informal sector, and working in the formal sector [4]. Additionally, a study on workers with disabilities was also conducted by Estika & Rumayya in 2024 aimed at analyzing the opportunities for disabled individuals to enter the labor market in Indonesia using logit regression [5]. Inclusive labor market participation of persons with disabilities in Indonesia, Denmark, France, and Ireland was also studied by Suwitanty and Setyari in 2024 using panel data regression fixed effect model [6]. The application of the SAE method to workers with disabilities has not yet been conducted, but the SAE method for estimating the school participation rate of people with disabilities has been carried out by Ridwansyah et al. in 2024 using the SAE HB Beta method to reduce the RSE value across all regions to below 25 percent [7]. Additionally, in other topics, the SAE HB method has been applied in research such as estimating the small area percentage of children aged 0–17 living below the poverty line [8], estimating household per capita consumption expenditure [9], and estimating the small area Unemployment Rate (TPT) at the sub-district level [10].

One of the HB methods utilizes Poisson-gamma distribution, which is capable to estimate the occurrence of a rare event in a small area. The simpler Poisson Regression Model is a basic model for count data, characterized by the assumption that the mean and variance are equal [11]. However, in count data, overdispersion is one of the limitations as it violates its assumption. Therefore, the Poisson-Gamma mixed model is frequently applied to address this issue [12]. SAE HB Poisson-Gamma does not assume that the data on the number of workers with disabilities follows a normal distribution. Instead, the data is assumed to follow a Poisson distribution with the average occurrence coming from a gamma distribution. Therefore, this research contributes to the implementation of SAE HB Poisson-Gamma in estimating the number of workers with disabilities in each province in Indonesia in 2024.

II. RESEARCH METHODS

2.1 Data Sources

This research utilizes secondary data sourced from two key datasets provided by Statistics Indonesia (BPS). The first dataset is obtained from the August 2024 Labor Force Survey (SAKERNAS), which is designed to estimate the employment status of workers with disabilities at national level. The number of workers with disabilities will be disaggregated to the provincial level as response variable (Y). The second dataset is obtained from the 2024 Village Potential Statistics (PODES) census, which provides auxiliary variables (X) at the village level. These complementary data sources enable a more comprehensive analysis of labor market outcomes for individuals with disabilities, as the latter provides valuable support for estimation at lower administrative levels.

There are four types of disability measurements based on the Washington Group Short Set (WG-SS) concept. The response variable in this research is defined by type III disability which is the classification adopted by the WG-SS. A person is considered to have a disability under type III if they respond with “a lot of difficulty” or “unable to do at all” to at least one of the relevant questions [13]. The WG-SS standard evaluates difficulties across six core universal activities: sight, hearing, mobility, self-care, cognition, and communication [14].

The selection of PODES variables in this research is adapted from the theoretical framework proposed by Goldfeld in 2015 and later enhanced by Fortune in 2022 about employment on people with disabilities [15]. These frameworks identify six key dimensions influencing employment outcomes for persons with disabilities: the socioeconomic environment, services, physical environment, social environment, governance, and urbanicity. These dimensions classify village-level characteristics that can be examined to understand their impact on employment accessibility and inclusion for populations with disabilities. Relevant variables presented in Table 3 are selected from the PODES dataset to represent some of these dimensions, ensuring that the analysis remains both theoretically grounded and in line with prior empirical research. Those variables are new as former studies focused in individual-level while this research uses provinces as analysis unit and deriving variables from Fortune's (2022) dimension is more suitable to get auxiliary variables' borrowing strength for the analysis method.

Table 3. Response Variable and Auxiliary Variable Candidates

Variable	Description
Workers with disabilities (Y)	Direct estimate of the number of workers with disabilities.
Sex Ratio (X_1)*	The number of men for every 100 women.
Healthcare Facilities (X_2)**	The number of healthcare facilities, including but not limited to hospitals, health centers, and clinics.
Economic Facilities (X_3)**	The number of economic facilities, including but not limited to markets, hotels, and restaurants.
Disabled Residents (X_4)*	The number of residents with disabilities.
Shackled Residents (X_5)*	The number of residents shackled.
Household Electricity (X_6)***	The number of households living on the electrical grid.
Small Businesses (X_7)*	The number of micro and small businesses operating.
Base Transceiver Stations (X_8)**	The number of base transceiver stations operating.
Primary Special Education Ratio (X_9)**	The number of villages with special education at primary level for every 10,000 residents.
Lower Secondary Special Education Ratio (X_{10})**	The number of villages with special education at lower secondary level for every 10,000 residents.
Upper Secondary Special Education Ratio (X_{11})**	The number of villages with special education at upper secondary level for every 10,000 residents.
Street Lights (X_{12})***	The number of villages with electrical street lights as primary street lighting.
Roads (X_{13})***	The number of villages with asphalt as primary roads.
Public Transits (X_{14})***	The number of villages with public transit connections.
Foreign Language Workshops (X_{15})**	The number of training centers specializing in foreign languages.
Computer Workshops (X_{16})**	The number of vocational institutions offering computer skills.
Tailoring Workshops (X_{17})**	The number of training centers specializing in tailoring and garment constructions.
Beauty Workshops (X_{18})**	The number of training centers specializing in beauty services.
Automotive Workshops (X_{19})**	The number of vocational institutions offering automotive skills.
Electronic Workshops (X_{20})**	The number of training centers specializing in electronics and electrical repairs.
Other Workshops (X_{21})**	The number of workshops/skill facilities specializing in other trade skills.

Dimensions [15]:
*) Socioeconomic Environment
**) Services

***) Physical Environment

2.2 Analysis Methods

The following methodological steps are undertaken to ensure robust and reliable small area estimates. These steps encompass both design-based and model-based approaches, beginning with direct estimation techniques and extending through model specification, diagnostics, and comparative evaluation using saeHB package on R software [16]. The process integrates classical survey estimation methods with hierarchical Bayesian techniques where applicable, thereby enhancing precision and inferential validity, particularly in domains with limited sample sizes. The detailed procedure is outlined as follows:

1. Horvitz-Thompson estimator is utilized to obtain direct estimates for each area [17].
$$\hat{Y}_{ht} = \Sigma_{i\epsilon s}\frac{y_i}{\pi_i}$$
Where s is the sample from a finite population π_i is the sampling fraction for observation y_i. Taylor linearization is applied to approximate the standard errors associated with the direct estimates.
2. The relationship between auxiliary variable candidates and the response variable is assessed through exploratory data analysis to identify potential variables that can be used for borrowing strength in small area models.
3. Stepwise Poisson model selection is performed to determine the most parsimonious set of covariates for the small area estimation model. Model diagnostic is also conducted to confirm the validity of the Poisson model, as it would be suitable for count data under the equal mean and variance assumption [18].
4. The presence of overdispersion is tested by deviance diagnostic. The Poisson-Gamma model may be a more suitable alternative for the small area model on overdispersed data.
5. Hierarchical Bayesian small area estimation is implemented with the corresponding Markov Chain Monte Carlo (MCMC) algorithms to derive the posterior distributions of the model parameters. The data model, or the likelihood function based on the Poisson-Gamma formulation, is presented below [19].
$$y_i|\theta_i \overset{iid}{\rightarrow} Poisson(e_i\theta_i)$$
where y_i is the observed cases, e_i is the expected cases, and θ_i is the relative risk as the parameter to be estimated. The relative risk itself is assumed
$$\theta_i|\alpha,\nu \overset{iid}{\rightarrow} Gamma(\nu,\alpha)$$
which parameters are linked with covariates. The shape parameter ν is shared across all areas, while $\alpha = \nu/\mu$ is the rate parameter with area-specific effect. The expected value is modelled through covariates using the following link function.
$$\log \mu_i = \boldsymbol{x}_i^T\boldsymbol{\beta} + u_i$$
where $\boldsymbol{x}_i^T$ is the vector of auxiliary variables and $\boldsymbol{\beta}$ is its estimated coefficient, and random effect u_i for capturing area-specific variation not captured by the covariates. The hyperparameter $\boldsymbol{\beta}$ is assumed to follow $N(0, \boldsymbol{\Sigma_\beta})$, while the variance of the random effect is following inverse gamma distribution.
6. The convergence of MCMC is diagnosed using statistical tools such as trace plots. Posterior summaries may be interpreted only if convergence is deemed satisfactory; otherwise, the number of iterations must be increased or the model must be respecified.

7. The small area estimation model is compared against the direct estimates to evaluate the performance. Relative standard errors are utilized as criteria to assess the improvement in precision offered by the indirect estimate approach.

III. RESULT AND DISCUSSION

3.1 Overview of Direct Estimates of Workers with Disabilities at the Provincial Level in Indonesia in 2024

Figure 1 illustrates the distribution of direct estimates of workers with disabilities across Indonesian provinces in 2024. The color gradient on the map, ranging from white to black, represents increasing numbers of workers with disabilities. Provinces on the island of Java, particularly West Java, Central Java, and East Java, record the highest figures, ranging from 100,000 to 1 million, as indicated by black color. This pattern in line with the high population density in these regions. In contrast, provinces with fewer than ten thousand workers with disabilities tend to be in less densely populated areas, such as Kalimantan, Maluku, and Nusa Tenggara.

Figure 1. Distribution of Disability Workers by Province in Indonesia, 2024

Based on the descriptive statistics in Table 4, the average number of workers with disabilities in Indonesia in 2024 is 24,538 people. The estimated number of workers with disabilities at the provincial level exhibits a wide variation, ranging from a minimum of 793 people in South Papua Province to a maximum of 155,928 in East Java Province.

Table 4. Summary of Direct Estimation Statistics of workers with disabilities at the Provincial Level in Indonesia

Descriptive Statistics	Direct Estimate	Relative Standard Error (%)
Min	793	7.76
Q1	6,052	13.35
Median	11,784	15.47
Mean	24,538	17.93
Q3	24,069	21.48

Max	155,928	40.46

In terms of estimation quality, the relative standard error (RSE) value varies from 7.76% to 40.46% in terms of estimation quality. An estimate is deemed to be reasonably precise if its RSE is less than 25%. In this instance, most provinces have accurate estimates because the median and first to third quartile values are below the cutoff. Nonetheless, several provinces have high RSE, such as Papua Barat (40.46%), Kepulauan Riau (30.29%), and Papua Barat Daya (29.81%). The result suggests that the estimation is less accurate and most likely takes place in places with a small population or sample. Therefore, it is necessary to use indirect estimation techniques like the Small Area Estimation (SAE) method for Indonesian workers with disabilities at the provincial level.

3.2 Estimation of the Number of workers with disabilities at the Provincial Level in Indonesia in 2024 with SAE HB Poisson

Indirect estimation begins with testing the normality of the direct estimator and selecting the auxiliary variables. The Shapiro-Wilk test is used to statistically test the normality of the direct estimator. The test results in Table 5 show that the p-value is less than 0.05, indicating that the direct estimator of workers with disabilities is not normally distributed. Therefore, the appropriate method for indirect estimation is SAE HB Poisson. This method does not assume a normal distribution in its direct estimator. In addition, SAE HB Poissonis suitable for data of a count nature with values greater than zero.

Table 5. Results of the Shapiro-Wilk Test for Direct Estimation of workers with disabilities

Test Statistics (W)	**p-value**	**Conclusion**
0.534	8.25×10^{-10}	Not Normally Distributed

From 21 candidate auxiliary variables, variable selection was carried out using stepwise regression with a direct estimator in the form of logit. The results of the stepwise regression indicate that three variables are suitable for parameter estimation using indirect estimation: sex ratio (X_1), Shackled Residents (X_5), and Computer Workshops (X_{16}). The multicollinearity examination, as presented in the Table, shows a VIF value of less than 10, indicating that there is no strong relationship between the auxiliary variables.

Table 6. Multicollinearity Examination

Auxiliary Variables	**Variance Inflation Factor**	**Conclusion**
Sex ratio	1.466	No Multicollinearity Occurs
Shackled Residents	1.183	
Computer Workshops	1.416	

Before conducting the estimation, it is essential to check the response variable, which is assumed to follow a Poisson distribution, for equidispersion. However, as observed in Figure 3, which illustrates the distribution pattern of the response variable represented by the number of workers with disabilities in Indonesia, it can be inferred that the data exhibits a right-skewed distribution. The high frequency of near-zero data and presence of a few very large data indicate a higher variance than its mean, suggesting the presence of overdispersion.

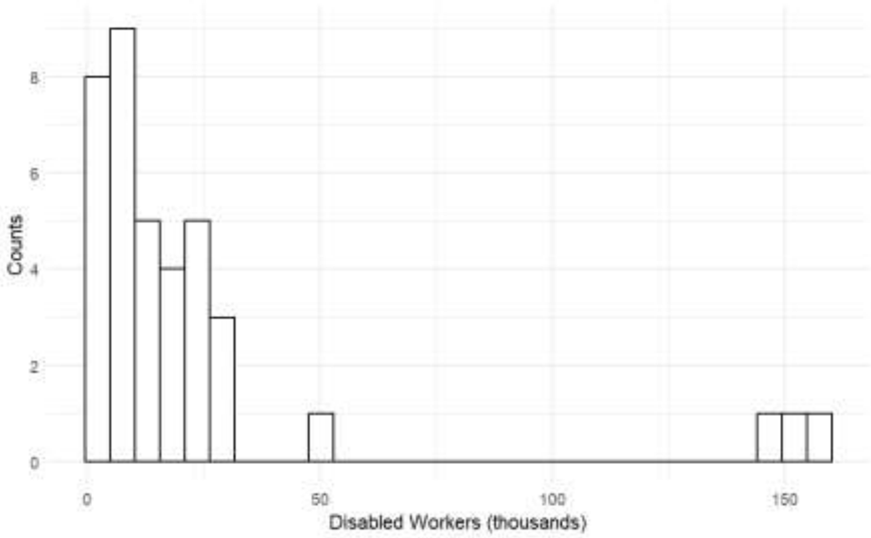

Figure 2. Frequency of workers with disabilities Across Provinces

Therefore, a more formal test was conducted using the dispersion ratio, Pearson's Chi-Squared statistic, and significance testing of the Chi-Squared statistic. If the dispersion ratio and Pearson's Chi-Squared values exceed 1, and the significance test of the Chi-Squared statistic rejects the null hypothesis, this indicates the presence of overdispersion. As shown in Table 7, both the dispersion ratio and Pearson's Chi-Squared results indicate values greater than 1, and the null hypothesis is rejected. Thus, it can be concluded that the data on the number of workers with disabilities in Indonesia exhibits overdispersion and the appropriate SAE HB method to address this overdispersion is the SAE HB Poisson-Gamma model.

Table 7. Overdispersion Test

Criterion	**Value**
Dispersion Ratio	4980.71
Pearson's Chi-Squared	169349.620
P-value	< 0.001

The SAE HB Poisson-Gamma method is incorporated into Bayesian inference using the Markov Chain Monte Carlo (MCMC) technique. By using 100 iterations, 20,000 MCMC iterations, 6,000 thinning intervals, and 2 burn-ins, the results show that the iteration process has converged, as indicated by the autocorrelation plot and density plot in Figure x. It can be seen that for all parameters, the autocorrelation plot has been cut off at the first leg. The trace plot does not exhibit a periodic pattern and features a density plot that resembles a bell curve and tends to be smooth.

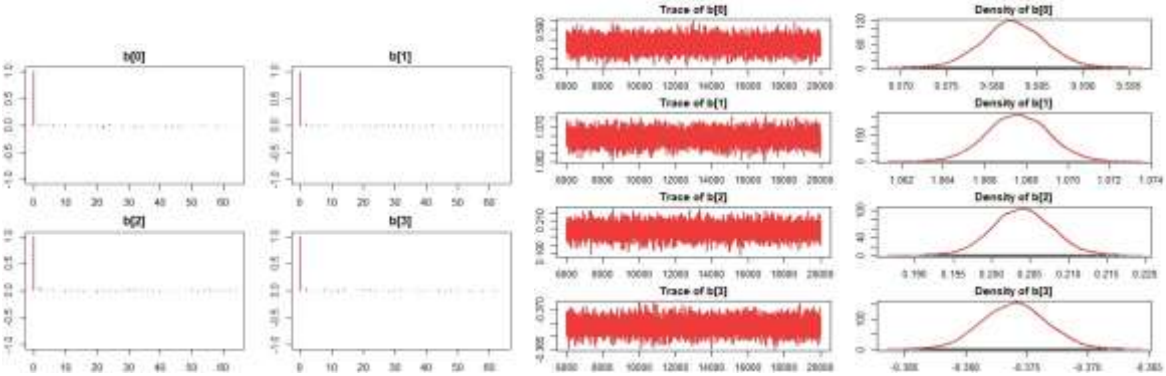

Figure 3. Autocorrelation, Trace, and Density Plot

After the iteration process has converged, an indirect estimate can be made of the number of workers with disabilities at the provincial level in Indonesia in 2024, using three selected associated variables.

Table 8. Estimated parameter coefficients of the SAE HB Poisson-Gamma model along with 95 percent credible intervals

Variable	Coefficients	Standard Deviation	Credible Interval	
			2.5%	97.5%
Intercept	9.582	0.0034	9.576	9.588
X_1	1.068	0.0015	1.065	1.071
X_5	0.204	0.0039	0.196	0.212
X_{16}	-0.376	0.0026	-0.381	-0.371

Based on the parameter estimation results in Table 8, it can be seen that within the credible interval range of 2.5 percent to 97.5 percent, all parameter estimation results do not contain zero values, indicating that all auxiliary variables included in the model are significant to the direct estimator.

From the results of the indirect estimation of parameters, an indirect estimate of workers with disabilities at the provincial level in Indonesia for 2024 has been obtained. The following table presents descriptive statistics of the results of the estimation of workers with disabilities using the direct estimation method and the Small Area Estimation HB Poisson-Gamma method in Table 9.

Table 9. Descriptive statistics of direct and indirect estimates of provincial-level workers with disabilities in Indonesia

Descriptive Statistics	Direct Estimate	SAE HB Poisson-Gamma
Min	793	794
Q1	6,052	6,054
Median	11,784	11,786
Mean	24,538	24,538
Q3	24,069	24,072
Max	155,928	155,963

Table 9 shows that the result of indirect estimation exhibit a pattern and value similar to those of the direct estimation. Starting from the minimum median value to the maximum value, there is no significant difference between the direct estimation results and the indirect estimation results. The average value of the estimation results between the two methods is the same, which is 24,538 workers with disabilities. The similarity of these statistical values shows that the HB Poisson-Gamma approach can replicate the distribution of direct estimates in aggregate but still offers precision advantages. It implies the bias of HB Poisson-Gamma is negligible because the validity is as accurate as direct estimator. However, the data variance is lower, which often inflated in small samples, resulting a better indicator in terms of precision.

3.3 Evaluation of Direct and Indirect Estimates of the Number of workers with disabilities at the Provincial Level in Indonesia in 2024

To evaluate the performance of both methods, it can be seen from the relative standard error (RSE) value of both methods. RSE is an important indicator to assess the accuracy or reliability of the estimate, the lower the RSE, the higher the level of confidence in the estimate. The following Table 10 presents a summary of RSE statistics for both approaches.

Table 10. Descriptive statistics of Relative Standard Error (RSE) from direct and SAE HB Poisson-Gamma estimates of provincial-level workers with disabilities in Indonesia (in percentage)

Descriptive Statistics	Direct Estimate	SAE HB Poisson-Gamma
Min	7.760	0.256
Q1	13.350	0.637
Median	15.465	0.918
Mean	17.929	1.096
Q3	21.478	1.289
Max	40.460	3.572

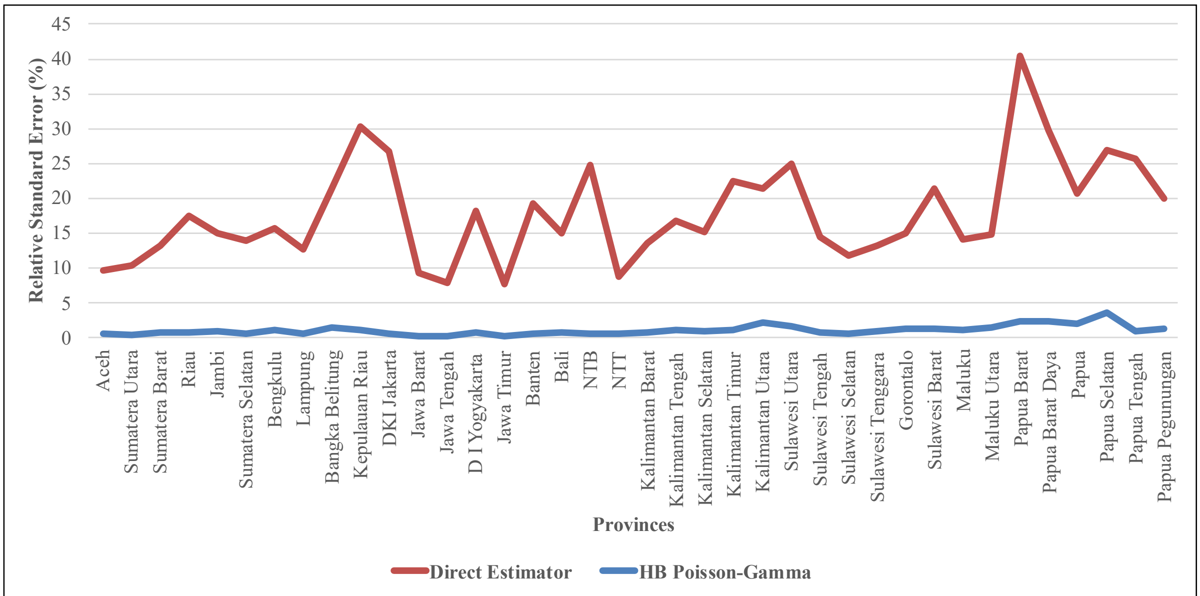

Figure 4. Direct estimation and HB Poisson-Gamma RSE by province in Indonesia in 2024

Based on Table 10, the direct estimation has a higher RSE value with an average of 17.93% and reaching a maximum of 40.46%, which indicates that there is still instability in the estimation, especially in areas with a small sample size. In contrast, the HB Poisson-Gamma method shows the best results with a very low RSE value with a median of 0.92%, an average of 1.09%, and a maximum value of 3.57%. This indicates that the method yields substantially more precise estimates compared to other methods. Thus, based on this RSE accuracy indicator, the SAE HB Poisson-Gamma method is the best method with the highest reliability in estimating the number of provincial workers with disabilities in Indonesia.

Figure 4 illustrates that, based on the orange line representing direct estimation, several provinces still exhibit Relative Standard Error (RSE) values exceeding 25 percent. In contrast, on the blue line it can be seen that the use of the HB Poisson-Gamma model has been able to reduce the RSE value to less than 25 percent for all provinces in Indonesia. This is also illustrated in the table of the number of provinces in Indonesia according to the estimation method and RSE category as follows:

Table 11. Number of provinces in Indonesia according to RSE estimation method and category

RSE Category	Direct Estimate	SAE HB Poisson-Gamma
Less than 25%	30	38
25% to 50%	8	-
More than 50%	-	-

Based on the results presented in Figure 4 and Table 11, the SAE HB Poisson-Gamma model has been able to estimate the number of workers with disabilities for all provinces in Indonesia, while also reducing the RSE value to below 25 percent. This indicates that the SAE HB Poisson-Gamma model is able to estimate the number of workers with disabilities better than the direct estimation method. Therefore, the HB Poisson-Gamma estimator will be used as a basis for mapping to see the picture of the number of workers with disabilities at the provincial level in Indonesia based on regional aspects.

3.4 Mapping Number of workers with disabilities at Provincial Level in Indonesia in 2024 Based on Best Estimate Results

To examine the number of workers with disabilities across provinces in Indonesia, a thematic choropleth map is presented. The number of workers with disabilities is grouped into three categories using the Natural Breaks method. Based on Figure 5, most provinces in Indonesia have a number of workers with disabilities in the range of 794 to 13,865, in this study the range of the number of workers is grouped as a low category. In more detail, from the results of grouping the number of workers with disabilities using the Natural Breaks method, 22 provinces are in the range of 794 to 13,865 workers (low category), 13 provinces are in the range of 13,865 to 48,845 workers (medium category), and 3 provinces are in the range of 48,845 to 155,963 workers (high category). The 3 provinces with a high number of workers with disabilities are all on the island of Java, namely West Java, Central Java, and East Java. Meanwhile, the 13 provinces that have a moderate number of workers with disabilities are mostly located on the islands of Bali, Nusa Tenggara, and Sumatra. There are only 2 provinces outside of these three islands, namely West Kalimantan province on the island of Kalimantan and South Sulawesi province on the island of Sulawesi. The other 22 provinces with a low number of workers with disabilities are mostly spread across the islands of Kalimantan, Sulawesi, Maluku, and Papua.

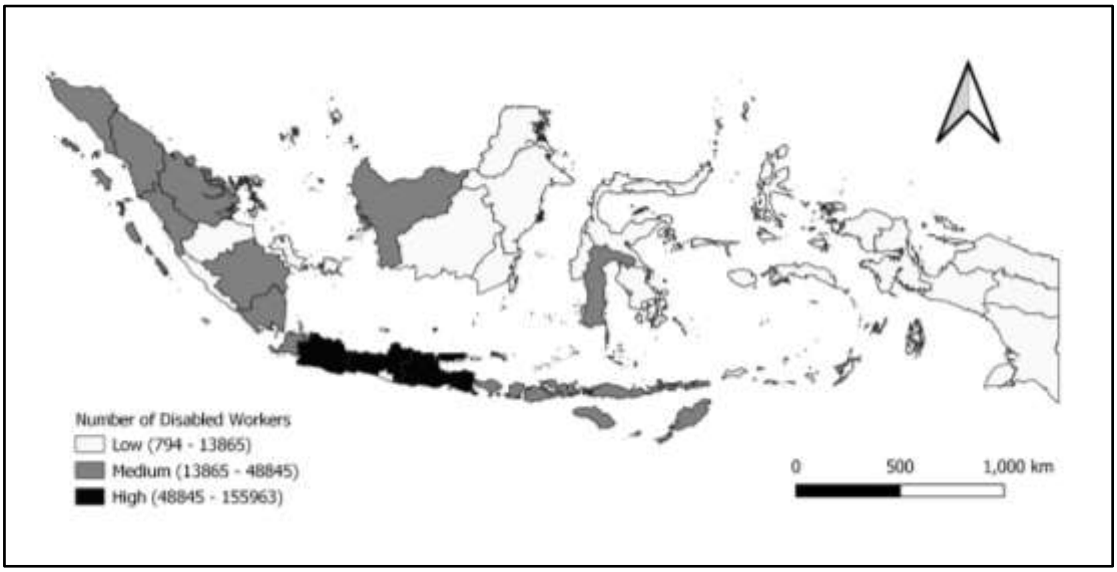

Figure 5. Choropleth map workers with disabilities in Indonesia in 2024 based on the HB Poisson-Gamma estimator

IV. CONCLUSION

The findings of this study show that only 30 out of 38 provinces in Indonesia have a Relative Standard Error (RSE) of less than 25 percent using direct estimation method. In other words, there are still 8 other provinces in Indonesia with high RSE, resulting in less precise estimates. The SAE HB Poisson-Gamma method, as part of the indirect estimation methods, proves to be the better method for estimating the number of workers with disabilities at the provincial level in Indonesia in 2024. This is indicated by the overall lower RSE of the HB Poisson-Gamma estimator compared to the direct counterpart. Additionally, the proposed HB Poisson-Gamma estimator has been shown to be reliable and valid.

The method used in this study resolves the issue of overdispersion in count data, particularly in estimating the number of workers with disabilities across provinces in Indonesia. This improves the precision of estimates, especially in regions with limited sample sizes. Based on the model results, the number of workers with disabilities within provinces in Indonesia is ranging between 794 and 13,865 individuals. That range is categorized as low in this study. However, there are still 3 provinces with a high number of workers with disabilities (ranging from 48,845 to 155,963 workers), most of which are located on the island of Java, specifically in West Java, Central Java, and East Java.

The availability of reliable statistics at provincial level enables the government to identify geographical disparities in labor force which are obscured by national aggregates. The lower-level estimation supports targeted allocation of employment and social security of certain demographic groups and allows monitoring of disability-related inclusive labor policies at sub-national levels. The proposed SAE approach provides evidence-based while remaining feasible design for labor policy decision-makers.

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