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MATHEMATICAL MODELING OF LEACHATE IN LANDFILLS: OVERVIEW AND APPLICATION

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Abstract. Jatibarang landfill plays a crucial role as a domestic waste treatment unit for the Semarang City area. Accumulated domestic waste generates leachate, which typically contains toxic chemicals, pathogenic bacteria, as well as organic and inorganic compounds. Uncontrolled management of domestic waste can lead to severe environmental impacts, such as the spread of diseases and the degradation of water quality. It is essential for management to monitor leachate concentration distribution to facilitate effective mitigation efforts. The phenomena and challenges associated with TPAs can be examined through mathematical modeling, particularly dynamic models to study leachate dispersion behavior. Numerical approaches, especially the finite difference method, play a significant role in solving models that are otherwise analytically intractable. This article reviews existing mathematical models for leachate dispersion and presents their application using a two-dimensional Forward Time Central Space (FTCS) scheme at Jatibarang landfill. Field measurements of cadmium (Cd) and copper (Cu) at five sampling sites were used to calculate the diffusion coefficient via Mean Squared Displacement (MSD), yielding $D = 3.66 \times 10^{-4}$ cm²/s. Results show cadmium disperses widely across all terminal sites, while copper remains concentrated near the source, highrisk zones are identified around site 3.

Keywords: leachate; mathematical models; landfill

I. INTRODUCTION

Sustainable Development Goal (SDG) 6 aims to achieve access to safe sanitation by 2030. However, the provision of adequate leachate collection and treatment facilities remains a major challenge, particularly in Southeast Asian countries [1][2][3][4]. Contamination and pollution are pressing global environmental issues, with severe impacts on human health, food production, economic growth, and natural ecosystems [5] [6]. The expansion of residential areas contributes to new problems, particularly related to waste disposal [7]. Urban development marked by economic growth, population increase, and urban expansion has become a major driver of environmental pollution[8] [9]. The denser a region becomes, the more complex the pollution issues, especially those caused by residential waste. If left unaddressed, the consequences of this pollution can be severe [10]. Uncontrolled leachate can contaminate soil and water bodies in Indonesia, particularly on the island of Java [11] [12]. Leachate pollution is often the result of improper disposal of domestic and industrial waste into the environment either directly or indirectly without adequate treatment. Numerous studies have highlighted the challenges involved in leachate management [13].

Understanding the distribution and treatment of leachate is a critical step toward ensuring that ecosystems surrounding waste processing sites such as rivers are safe for reuse in various applications, including agricultural irrigation, street cleaning, fire control, geothermal energy production, industrial processes, commercial washing, and construction [14]. One of the key prerequisites for effective leachate management is the ability to estimate its distribution and concentration at various points around landfill sites, thereby enabling the identification of high-risk areas more accurately [15].

This paper focuses on identifying the spatial distribution and key challenges in leachate treatment, with an emphasis on mathematical modeling approaches aimed at optimizing the understanding of leachate concentration patterns while minimizing their environmental impact. The article is systematically structured to provide a comprehensive understanding of the topic. The introduction outlines the research background, highlighting the importance of leachate management in supporting environmental sustainability and its relevance to the achievement of SDG 6. Furthermore, the introduction presents the main objective of the study, which is to review existing mathematical models for leachate dispersion and to apply the Finite Difference Method (FDM) specifically the Forward Time Central Space (FTCS) scheme to simulate the spatial distribution of cadmium (Cd) and copper (Cu) leachate at Jatibarang landfill, Semarang[16], [17]. The diffusion coefficient is determined using Mean Squared Displacement (MSD) derived from field concentration data, and simulation results are visualized through contour mapping to identify high-risk contamination zones. The structure of the article includes a literature review on leachate profiles and components, a review of existing leachate dispersion models and their limitations, the application of FDM to Jatibarang landfill with field data, and conclusions that underscore the implications of the research and its potential future applications.

II. STUDY RESULT

2.1 Overview of the Jatibarang landfill

The Jatibarang landfill is one of the centralized waste treatment facilities located in Semarang City. Situated in Kedungpane Village, Mijen District, Semarang City, the site covers an area of approximately 46.1830 hectares. Jatibarang is the largest landfill site in Central Java and serves as the primary waste disposal facility for Semarang, accommodating approximately 70% of the city's total waste. The site has the capacity to handle up to 800 tons of waste per day [18]. Although Jatibarang landfill is officially managed using a sanitary landfill system, its implementation does not fully comply with the established standards for sanitary landfill operations. This deviation is primarily due to seasonal limitations: soil coverage for waste compaction can only be performed during the dry season, while during the rainy season, it becomes exceedingly difficult to cover waste after it reaches a certain height. The landfill is currently operated using an open dumping method rather than a modified controlled landfill system [19].

Located near residential areas, Jatibarang landfill is one of the main sources of waste pollution in Semarang. The composition of waste entering the site consists of approximately 61.95% organic waste and 38.05% inorganic waste. The generated leachate is channeled into storage ponds for further treatment. However, this method has several limitations, the most critical of which is the landfill’s overcapacity. The volume of leachate produced often exceeds the system's treatment capacity, leading to a significant risk of contamination to soil, groundwater, and nearby rivers [11].

Consequently, landfill leachate has become a significant environmental concern. Previous studies reported that leachate generated at Jatibarang landfill contains elevated concentrations of heavy metals, particularly cadmium (Cd) and copper (Cu), which may threaten surrounding soil and groundwater quality [11], [16]. Heavy metals are among the most hazardous components of landfill leachate because they are non-biodegradable, persistent in the environment, and capable of bioaccumulating within biological systems [17], [18]. Field measurements conducted at five monitoring sites around the landfill recorded cadmium concentrations ranging from 0.060 to 0.098 mg/L, with four out of five sites exceeding the TCLP-C regulatory threshold of 0.06 mg/L established by the Indonesian Ministry of Environment and Forestry [19]. These findings indicate a substantial risk of environmental contamination and highlight the urgent need for spatially explicit modeling of leachate dispersion to support evidence-based mitigation and landfill management strategies.

Field observations conducted at Jatibarang landfill identified five leachate sampling sites based on topographic conditions, where leachate was assumed to flow from higher to lower elevation points. site 3, located at the natural leachate pond, was identified as the primary leachate source, while site 1, 2, 4, and 5 served as terminal points of leachate dispersion. The site was built directly on natural soil without an impermeable liner system, with a predominantly sandy clay soil texture, which confirms that leachate transport at this site is governed primarily by diffusion rather than advection. Table 1 presents the leachate heavy metal concentrations measured at each sampling site.

Table 1. Heavy Metal Concentrations of Leachate at Jatibarang landfill Sampling sites

Site	Leachate Flow	Cd (mg/L)	Regulatory Cd Status (TCLP-C Limit)	Cu (mg/L)	Regulatory Cu Status (TCLP-C Limit)
1	Dispersion 1 (Site 3 to Site 1)	0.060	Threshold	0.037	Below
2	Dispersion 2 (Site 3 to Site 2)	0.081	Exceeds	0.093	Below
3	Leachate source	0.098	Exceeds	0.626	Below
4	Dispersion 3 (Site 3 to Site 4)	0.092	Exceeds	0.090	Below
5	Dispersion 4 (Site 3 to Site 5)	0.0917	Exceeds	0.098	Below

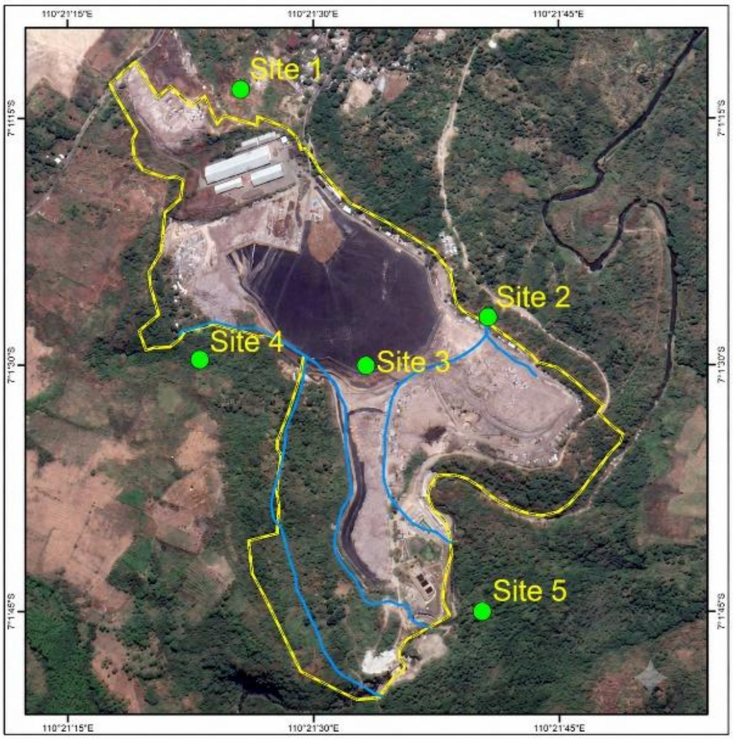


Figure 1. Jatibarang Landfill Site, Semarang City

2.2 Characteristics of Leachate

Leachate is a liquid generated when water primarily from rainfall percolates through accumulated waste at a final processing site, dissolving various chemicals, organic compounds, heavy metals, and microorganisms in the process [20]. In developing countries across Asia and Africa, leachate typically exhibits higher concentrations of Chemical Oxygen Demand (COD) and Biological Oxygen Demand (BOD), largely due to the presence of unsorted organic matter in household waste [21],[22]. The composition of leachate is heavily influenced by seasonal variation. During the dry season, leachate tends to be more concentrated, with significantly higher levels of COD and Total Suspended Solids (TSS) compared to that produced during the rainy season [23][24].

Key components of leachate include suspended solids, biodegradable organic matter, pathogens, and nutrients [25]. One of the most notable characteristics of leachate is its high organic content, which often includes heavy metals [26], nitrogen and phosphorus compounds [27][28], as well as bacteria, algae, zooplankton, detrital organic matter, ammonia (NH_3), dissolved oxygen (DO), total coliform, fecal coliform, and biochemical oxygen demand (BOD) [29]. When leachate exceeds treatment capacity and is not properly managed, it poses serious public health risks, including the spread of waterborne diseases such as hepatitis, cholera, and typhoid (Pan et al., 2019). Among the most hazardous components of leachate are heavy metals persistent, toxic pollutants that are non-biodegradable and can bioaccumulate in living organisms [30]. The presence of heavy metals poses a significant threat to water quality and human health, with the potential to cause severe illnesses [30][31]. A major source of heavy metal contamination is the direct discharge of waste into drainage systems or rivers, which increases toxicity risks. Research has shown that heavy metals are predominantly found in suspended particulate matter within waste streams [32]. It is therefore essential for landfill operators to monitor and map the spatial distribution of leachate concentrations within and around the landfill site. Accurate mapping of leachate concentration levels can greatly assist in the development of appropriate mitigation and rehabilitation strategies tailored to the specific distribution patterns observed.

2.3 Leachate Dispersion through Mathematical Modeling Approaches

Mathematical modeling plays a crucial role in understanding and predicting the dispersion of pollutants, such as leachate, in Final Processing Sites. The modeling of pollution dispersion using diffusion equations has become a central focus in environmental studies. These models are employed to solve partial differential equations that describe the transport of contaminants through various media. A substantial body of literature has proposed and applied various mathematical models to simulate leachate dispersion behavior in landfills. These models can be broadly categorized based on their governing equations, dimensionality, and the physical processes they represent. Table 2 summarizes the principal models reviewed in this study, including their roles and identified weaknesses.

Table 2. Summary of Existing Mathematical Models for Leachate Dispersion

Model	Role	Weakness
Advection–Dispersion Equation (ADE) [33]	Simulates contaminant transport through advection and hydrodynamic dispersion in porous media.	Assumes homogeneous porous media and equilibrium transport; less accurate in highly heterogeneous landfill environments.
HELP Model [34]	Estimates leachate generation using water-balance components (rainfall, runoff, evapotranspiration, drainage).	Focuses on leachate quantity rather than contaminant concentration distribution.
MODFLOW / MT3DMS [35]	Three-dimensional groundwater flow and contaminant transport simulation.	Requires extensive hydrogeological data and calibration.
HYDRUS Model [36]	Simulates variably saturated flow and multi-component solute transport in 1D/2D/3D domains.	Highly sensitive to hydraulic parameters and computationally intensive.
Dual Continuum (Dual-Domain) Model [37]	Represents preferential flow through macropores and matrix exchange.	Difficult parameterization; requires detailed field measurements.
Coupled Kinetic Model [38]	Simulates pollutant release and transport coupled with biodegradation, settlement, and oxygen dynamics.	Complex calibration and large number of parameters.
Stochastic Model [39]	Incorporates uncertainty and variability in pollutant transport processes.	Strong dependence on probability distributions and field validation data.
Finite Difference Method (FDM)[40] [41]	Numerical solution of diffusion-advection equations with simple implementation.	Numerical dispersion and grid dependency may reduce accuracy.

Overall, each modeling approach offers specific strengths suited to particular aspects of leachate dispersion analysis, but none provides a universally adequate solution for all conditions encountered in real landfill systems. The ADE model provides a tractable theoretical foundation but struggles with heterogeneous MSW media. Richards' equation is well-suited for unsaturated zone flow but cannot capture preferential flow paths. The HELP model is practical for volume estimation but provides no spatial contamination information. MODFLOW/MT3DMS offers three-dimensional simulation capability but demands extensive data and technical expertise. HYDRUS delivers multi-component transport simulation but is highly parameter-sensitive. The dual continuum model improves physical realism but significantly increases calibration complexity. The coupled kinetic model is the most comprehensive physically but is practically unrealizable in full form. Stochastic models quantify uncertainty effectively but require long-term monitoring data rarely available in developing-country settings.

This review therefore highlights that for TPA settings such as Jatibarang where data availability, instrumentation, and technical capacity may be constrained the Finite Difference Method (FDM) applied to diffusion-based mathematical models represents a practical, physically meaningful, and computationally accessible approach. Its ease of implementation and compatibility with the relatively simple geometric structures of landfill sites, as demonstrated by multiple studies reviewed in this article, position FDM as the most appropriate numerical method for advancing leachate dispersion modeling at Indonesian TPAs.

2.31. Application of the Diffusion-Based FDM Model at Jatibarang landfill

Building upon the reviewed models, this study applies a simplified diffusion-based model to Jatibarang landfill a natural landfill where advection can be disregarded due to the absence of a liner system and the dominance of passive diffusion in leachate transport. The governing equation adopted is the two-dimensional diffusion equation:

$$\frac{\partial \mathrm{C}}{\partial \mathrm{t}} = D\left(\frac{\partial^2 C}{\partial \mathrm{x}^2} + \frac{\partial^2 C}{\partial \mathrm{y}^2}\right)$$

where $C(x, y, t)$ denotes the concentration field as a function of space and time, and $\boldsymbol{D}$ is the diffusion coefficient ($D > 0$). This simplification is justified by the site conditions at Jatibarang: the landfill operates as an open dumping system without a formal liner, and field observations confirm that leachate migrates primarily through passive diffusion within a sandy clay soil layer. The FTCS (Forward Time Central Space) scheme is used to discretize the equation, yielding:

$$C_{i,j}^{n+1} = C_{i,j}^{n} + \alpha\left(C_{i+1,j}^{n} - 2C_{i,j}^{n} - C_{i-1,j}^{n}\right) + \beta(C_{i,j+1}^{n} - 2C_{i,j}^{n} - C_{i,j-1}^{n})$$

where $\alpha = \frac{D\Delta t}{\Delta x^2}$, $\beta = \frac{D\Delta t}{\Delta y^2}$. The diffusion coefficient D was determined from field data using Mean Squared Displacement (MSD):

$$D = \frac{p^2}{4\Delta \mathrm{t}}$$

where p^2 is the squared displacement from the source (site 3) to each terminal site, and Δt is the travel time. Table 1 presents the calculated diffusion coefficients for each dispersion pathway.

Mathematical modeling provides both spatial and temporal analysis of leachate distribution. This approach directly addresses the limitations identified in the reviewed models, the FDM diffusion model presented here requires only field concentration measurements and spatial coordinates making it practically implementable under the data constraints typical of Indonesian TPA facilities. Therefore, the reviewed studies highlight the importance of integrating theoretical approaches with simplified numerical simulations to deliver more reliable yet applicable predictions of leachate dispersion in unlined landfills

2.4 Numerical Approaches for Solving Mathematical Models

Numerical methods are widely used to solve mathematical models, particularly differential equations that are difficult or impossible to solve analytically. These difficulties often arise due to the complexity of the model, such as the presence of nonlinear terms [42]. Various numerical techniques can be employed, including the finite difference method, finite element method, finite volume method, Runge-Kutta methods, among others. This paper focuses specifically on the application of the finite difference method (FDM) to Final Processing Sites (TPAs). The main advantage of the finite difference method over other numerical techniques lies in its ease of implementation and its compatibility with the relatively simple geometries often found at landfill sites. Several studies have demonstrated the application of the finite difference method in modeling leachate behavior at TPAs. For example, FDM has been used to simulate contaminant migration from leachate into protective soil layers, enabling more accurate analysis of ion migration in landfill liners [43]. [44] developed a corrected finite difference scheme to improve the accuracy of contaminant transport predictions involving diffusion and reaction in soil. [45] formulated a one-dimensional finite difference model for contaminant transport in unsaturated soil media, taking into account convection and dispersion processes. [46] created a numerical model to simulate leachate generation and transport in TPAs using a finite difference approach to predict environmental impacts. [47] applied GMS software, based on the finite difference method, to simulate contaminant transport in groundwater from inactive landfills. [48] developed a mathematical model to trace leachate plumes from TPAs into surrounding groundwater, utilizing FDM to predict the spatial spread of contaminants. [49] proposed a two-dimensional, dual-domain model using FDM to simulate leachate formation and water flow within TPAs. [50] developed a coupled kinetic model for simulating pollutant release and transport in deformable landfills, incorporating various physical and chemical processes. [51] conducted a parametric sensitivity analysis of leachate transport simulations in TPAs, using FDM to evaluate the influence of model parameters on simulation results. [52] applied a Galerkin-based model for groundwater transport, employing finite difference techniques to solve contaminant transport equations. Due to its robust capability in addressing diffusion-based transport phenomena, the finite difference method has become a valuable tool for data-driven decision-making in sustainable TPAs management. Future developments may involve integrating FDM with optimization algorithms, machine learning techniques, or other hybrid approaches to enhance both computational efficiency and prediction accuracy.

2.5 Simulation Results: Leachate Dispersion Patterns at Jatibarang landfill

The diffusion coefficient for each dispersion path is as follows: dispersion 1 is the calculation from Site 3 to Site 1, dispersion 2 is from Site 3 to Site 2, dispersion 3 is from Site 3 to Site 4, and dispersion 4 is from Site 3 to Site 5. In calculating the coefficient, a map of sample points that has been converted into a grid format is required. To find the diffusion coefficient, the following equation can be used:

$$D = \frac{p^2}{4\Delta \mathrm{t}}$$

defined as

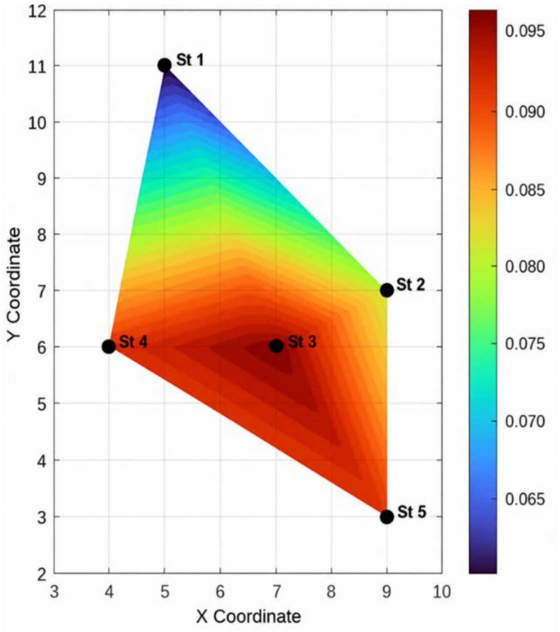
$$p^2 = (x - x_0)^2 + (y - y_0)^2$$

where (x_0, y_0) is the position of the pollutant source, $\Delta \mathrm{t}$ is the time difference (in hours) from Site 3 to the other Sites.
The results of calculating the leachate diffusion coefficient (*D*) at the Jatibarang Landfill are provided in Table 3.

Table 3. Diffusion Coefficient Values for Each Dispersion

Leachate Flow	Diffusion Coefficient (cm^2/s)
Dispersion 1 (Site 3 to Site 1)	5.03×10^{-4}
Dispersion 2 (Site 3 to Site 2)	3.47×10^{-4}
Dispersion 3 (Site 3 to Site 4)	3.13×10^{-4}
Dispersion 4 (Site 3 to Site 5)	3.01×10^{-4}

The diffusion coefficient estimated using the Mean Squared Displacement (MSD) approach yielded an average value of $D = 3.66 \times 10^{-4}$ cm^2/s. This value is considerably higher than the molecular diffusion coefficients commonly reported for contaminants in wastewater ($\sim 10^{-5}$–10^{-6} cm^2/s), indicating that pollutant transport at the Jatibarang Landfill is strongly influenced by soil heterogeneity and landfill structural characteristics [53]. The obtained coefficient confirms that leachate migration at the study site is predominantly governed by diffusion processes.



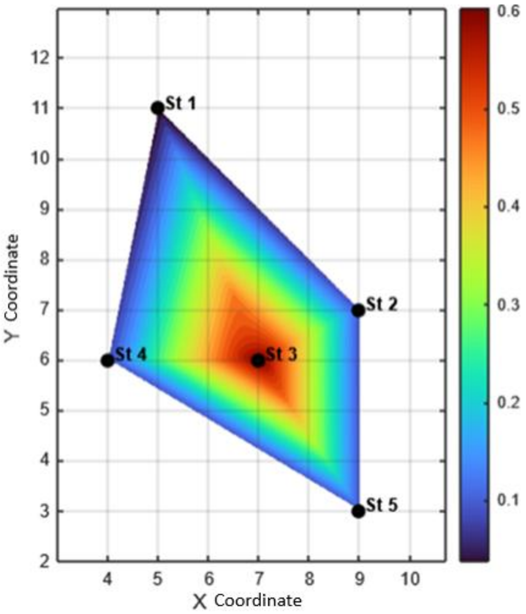


Figure 2. Cadmium Concentration Dispersion Pattern

Figure 3. Copper Concentration Dispersion Pattern

The simulation results revealed distinct transport behaviors between Cadmium (Cd) and Copper (Cu). Cadmium exhibited extensive spatial dispersion from the primary contamination source at site 3 toward all observation sites. Concentration contour mapping showed that Cd concentrations across most of the study area exceeded the TCLP-C regulatory threshold of 0.06 mg/L, indicating a high-risk contamination zone. This finding is consistent with previous studies reporting that Cd possesses relatively high environmental mobility and can migrate over broader spatial scales compared with other heavy metals [54] [55] [56].

In contrast, Copper (Cu) displayed a localized distribution pattern concentrated around site 3. Although the highest Cu concentration was detected at the source location, concentrations decreased rapidly with distance and remained well below the TCLP-C regulatory limit of 4.0 mg/L. This behavior agrees with observations reported by [57] [58]), who demonstrated that Cu contamination tends to remain confined near its source because of its relatively low mobility in soil environments. Consequently, the site 3 – site 5 corridor was identified as the priority area for environmental monitoring and remediation efforts, particularly with respect to Cadmium contamination. Overall, the modified finite difference model successfully captured the contrasting dispersion characteristics of Cd and Cu and provides a useful framework for leachate risk assessment and management at Jatibarang landfill.

III. CONCLUSION

The application of FDM at Jatibarang landfill demonstrates that cadmium (Cd) disperses widely from site 3 to all terminal sites, with over 80% of the observation area exceeding the TCLP-C threshold of 0.06 mg/L, indicating high contamination risk. Copper (Cu), in contrast, remains concentrated near the leachate source, with all sites well below the 4.0 mg/L TCLP-C standard. The average diffusion coefficient $D = 3.66 \times 10^{-4}$ cm²/s confirms that leachate transport at this site is diffusion-dominated. These findings provide a practical, data-driven foundation for risk zone mapping and leachate management planning at Jatibarang landfill and similar landfill sites in Indonesia.

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