

HOW TO COMBINE VAM AND DIJKSTRA'S ALGORITHM

Mizan Ahmad^{1*}, Riski Aspriyani², Eka Susilowati³

^{1,2,3} Universitas Nahdlatul Ulama Al Ghazali Cilacap

Email: ¹mizan.ahmad36@gmail.com, ²rizky.asp@gmail.com, ³eka250@gmail.com

*Corresponding author

Abstract. Solving transportation problems sometimes does not only require using one method or algorithm. Sometimes it is necessary to use several methods or algorithms at once. In this research, combining the Vogel's Approximation Method (VAM) and Dijkstra algorithm can be carried out if three assumptions are met. These three assumptions are based on the characteristics of each VAM and Dijkstra's algorithm, as well as the compatibility between the two.

Keywords: Transportation, combination of methods, VAM, Dijkstra algorithm.

I. INTRODUCTION

The transportation problem with m origins that contain various amounts of commodity must be shipped to n destinations to meet demand requirements. Specifically, origin i contain with amount a_i , and destination j has a requirement of amount b_j . It is assumed that the system is balanced in the sense that total supply equals total demand. That is

$$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j.$$

The numbers a_i and b_j , $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$ are assumed to be non-negative, and in many applications they are in fact non-negative integers. There is a unit cost c_{ij} associated with the shipping of the commodity from origin i to destination j . The problem is to find the shipping pattern between origins and destinations that satisfies all the requirements and minimizes the total shipping cost [1].

Based on Dantzig and Thapa [2], the classical transportation problem is to determine an optimal schedule of shipments that:

1. originate at sources where known quantities of a commodity are available;
2. are allocated and sent directly to their final destinations, where the total amount received must equal the known quantities required;
3. exhaust the supply and fulfill the demand; hence, total given demands must equal total given supplies;
4. and, finally, the cost of each shipment from a source to a destination is proportional to the amount shipped, and the total cost is the sum of the individual costs; i.e., the costs satisfy a linear objective function.

The transportation model is always balanced (sum of the supply = sum of the demand). If the model is unbalanced, a dummy source or a dummy destination must be added to restore balance [3]. One of the most popular methods for solving transportation problems is Vogel's Approximation Method (VAM). Vogel's Approximation Method (VAM) is a method used to solve transportation problems with the aim of minimizing distribution costs. The VAM method has been popular because it turns out that in practical applications it often finds a solution that is close to the optimal [2]. Some of the applications of VAM include the distribute raw fabric to several different locations at low cost and as optimally as possible [4] and optimization of distribution cost of concrete iron materials [5]. There are many other studies related to distribution optimization using the VAM method ([6], [7], [8], [9], [10],[11], [12], [13], [14], [15]).

According to Dantzig and Thapa [2], the shortest route problem is that of finding the minimum total "distance" along paths in an undirected connected network from the source $s = 1$ to the destination $t = m$. The distance can be actual miles, the cost or time to go between nodes, etc. One of the famous shortest route problem algorithms is the Dijkstra algorithm. Dijkstra's Algorithm is designed to determine the shortest routes between the source node and every other node in the network [3]. Some of the applications of Dijkstra algorithm include determine the shortest route from the city center to historical sites [16], determine disaster relief distribution solutions [17], determine the shortest route for travel [18], and determine the shortest path search futsal field location [19]. There are many other studies related to route optimization using the Dijkstra algorithm ([20], [21], [22], [23], [24], [25]).

In transportation problems, the Vogel's Approximation Method (VAM) and the Dijkstra algorithm are two techniques that are often used to solve different but equally important problems. These two methods, although used in different contexts, have the same goal, namely for optimization solutions. But some cases cannot be solved using either of them. Sometimes it is necessary to combine both or combine several methods or algorithms. This research aims to combine VAM and the Dijkstra algorithm in determining the optimal distribution route.

II. Combination of VAM and Dijkstra's algorithm

2.1. Vogel's Approximation Method (VAM)

According to Taha [3], VAM is an improved version of the least cost method that generally, but not always, produces better starting solutions.

1. For each row (column), determine a penalty measure by subtracting the smallest unit cost element in the row (column).
2. Identify the row (column) with the largest penalty. Break ties arbitrarily. Allocate as much as possible to the variable with the least unit cost in the selected row or column. Adjust the supply and demand, and cross out the satisfied row or column. If a row and a column are satisfied simultaneously, only one of the two is crossed out, and the remaining row (column) is assigned zero supply (demand).
3. (a) If exactly one row or column with zero supply or demand remains uncrossed out, stop.
(b) If one row (column) with positive supply (demand) remains uncrossed out, determine the basic variables in the row (column) by the least cost method. Stop.

- (c) If the all uncrossed out rows and columns have (remaining) zero supply and demand, determine the zero basic variables by the least cost method. Stop.
- (d) Otherwise, go to Step 1.

2.2. Dijkstra Algorithm

According to Taha [3], Dijkstra Algorithm: Let u_i be the shortest distance from node 1 to node i , and define $d_{ij} (\geq 0)$ as the length of arc (edge) (i, j) . Then the algorithm defines the label for an immediately succeeding node j as

$$[u_j, i] = [u_i + d_{ij}, i], d_{ij} \geq 0.$$

The label for the starting node is $[0, -]$, indicating that the node has no predecessor. Node labels in Dijkstra's Algorithm are of two types: temporary and permanent. A temporary label is modified if a shorter route to a node can be found. If no better route can be found, the status of the temporary label is changed to permanent.

Step 0. Label the source node (node 1) with the permanent label $[0, -]$. Set $i = 1$.

Step i. (a) Compute the temporary labels $[u_i + d_{ij}, i]$ for each node j that can be reached from node i , provided j is not permanently labeled. If node j is already labeled with $[u_j, k]$ through another node k and if $u_i + d_{ij} < u_j$, replace $[u_j, k]$ with $[u_i + d_{ij}, i]$.
(b) If all the nodes have permanent labels, stop. Otherwise, select the label $[u_r, s]$ having the shortest distance ($= u_r$) among all the temporary labels (break ties arbitrarily). Set $i = r$ and repeat step i .

2.3. VAM-Dijkstra Graph Model and Assumptions

The basic idea of combining the two methods is based on the following case: A company has several production sites which also act as warehouses ($G_1, G_2, \dots, G_n$) in several different areas. Each production site (warehouse) has its capacity (production or storage), its distribution area (independent), and total needs. If there are several production sites (warehouses) that have a capacity that exceeds requirements and there are also production sites (warehouses) that have a capacity that is less than required, then the production site (warehouse) with a capacity that exceeds requirements needs to send some of the excess production to the production site (warehouse).) with a capacity that is less than required. This distribution problem between warehouses can be solved using the transportation problem (in the case below, the VAM can be used) because the distribution considers capacity and costs. Furthermore, if the production place (warehouse) has several sales (marketing) agents each and the production place (warehouse) must send to each sales agent, then it can be solved using the shortest route problem (in the case below, the Dijkstra algorithm can be used) with the warehouse as the starting point because the distribution process begins with starting point (warehouse) to all sales agents and only considering distribution costs.

Thus, to solve the problem, a combination of the VAM and Dijkstra's algorithm can be used. Before combining the two, it is necessary to determine the models and assumptions required. First, its necessary to create a graph model of the problem. Let $G_1, G_2, \dots, G_k$ are warehouse, for $k \in \mathbb{N}$ and G_{in_i} are the agent of warehouse G_i , for $1 \leq i \leq k$ and $n_i \in \mathbb{N}$, following

is the graph model.

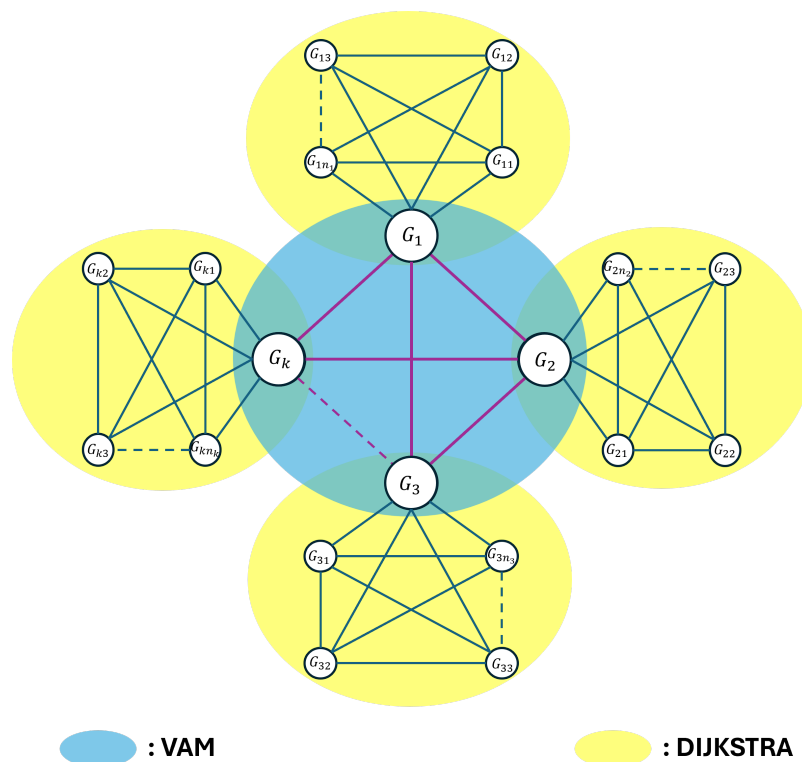


Figure 1. VAM-Dijkstra Graph Model

Based on the problem above and the graph model, to be able to combine the VAM and Dijkstra's algorithm, the following assumptions are required:

1. There is at least one warehouse that has excess capacity (excess production/more than the amount needed in the area) is called supplier and one warehouse that lacks capacity (underproduction/less than the amount needed in the area), is called recipient.
2. The transportation model (between warehouse) is always balanced (sum of the supply = sum of the demand). If the model is unbalanced, a dummy source or a dummy destination must be added to restore balance.
3. For each warehouse (supplier and recipient) has its own delivery area (independent delivery area) and becomes the root vertex (for Dijkstra Algorithm) in its own delivery area.

Next, to find a solution to this problem, simply use the VAM to determine the minimum cost solution for distribution between warehouses and use the Dijkstra algorithm to determine the shortest distribution route solution from each warehouse to each agent from that warehouse. Below are the steps:

1. For all G_i for $i = 1, 2, \dots, k$, determine the distribution route solution using VAM. This aims to meet the needs (demands) of each Warehouse.

2. Calculate C_w , which is the minimum distribution cost between Warehouses.
3. For each $i = 1, 2, \dots, k$, determine the distribution route from G_i to each G_{in_i} using the Dijkstra algorithm for each $j = 1, 2, \dots, n_i$.
4. Calculate C_{w_i} , which is the minimum distribution cost from G_i to each G_{in_i} .
5. Calculate C , which is the total distribution cost with

$$C = C_w + \sum_{i=1}^k C_{w_i}.$$

III. Implementation of VAM and Dijkstra's Algorithm Combination

3.1. Example Case

The following is an example of a case that can be solved using the combination of VAM and Dijkstra's algorithm.

A Company X has 4 production sites which are also warehouses. Each production site (warehouse) has a different production capacity (warehouse capacity). The following are details of the production quantities at each production site.

Table 1. The Information of Warehouse

Warehouse	Production Amount	Amount Needed	Supply	Demand
G_1	25	10	$25-10=15$	-
G_2	30	25	$30-25=5$	-
G_3	5	15	-	$15-5=10$
G_4	10	20	-	$20-10=10$

The following are distribution costs between warehouses.

Table 2. Distribution Cost (\$/item) between Warehouse

	G_1	G_2	G_3	G_4
G_1	-	2	5	7
G_2	2	-	8	4
G_3	5	8	-	5
G_4	7	4	5	-

Warehouse G_1, G_2, G_3, G_4 each have 2, 5, 3, and 4 agents. The following are distribution cost for each trip between warehouse and its agents.

Table 3. Distribution Cost (\$) for each trip between Warehouse and Its Agents

	G_1	G_{11}	G_{12}
G_1	-	4	9
G_{11}	4	-	3
G_{12}	9	3	-

	G_2	G_{21}	G_{22}	G_{23}	G_{24}	G_{25}
G_2	-	2	13	8	10	7
G_{21}	2	-	11	3	4	9
G_{22}	13	11	-	9	10	4
G_{23}	8	3	9	-	7	4
G_{24}	10	4	10	7	-	8
G_{25}	7	9	4	4	8	-

	G_3	G_{31}	G_{32}	G_{33}
G_3	-	3	4	8
G_{31}	3	-	5	3
G_{32}	4	5	-	6
G_{33}	8	3	6	-

	G_4	G_{41}	G_{42}	G_{43}	G_{44}
G_4	-	8	7	6	2
G_{41}	8	-	10	9	7
G_{42}	7	10	-	3	3
G_{43}	6	9	3	-	5
G_{44}	2	7	3	5	-

The company wants to distribute all its production results to all agents at minimum cost.

3.2. Solution

The following is a graph model for the problem.

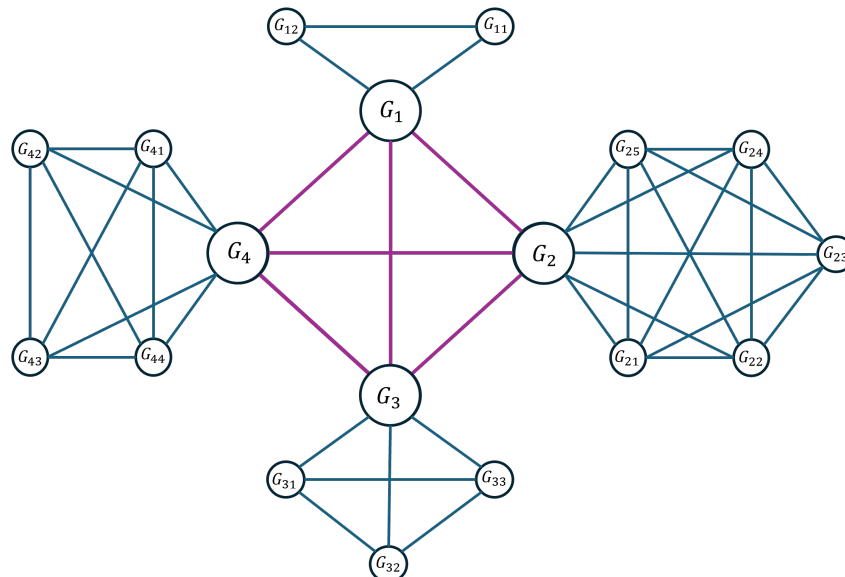


Figure 2. Graph Model

Based on the information above, its obtained:

1. There are two warehouses that has excess capacity (G_1, G_2) and two warehouses that lacks capacity (G_3, G_4).

2. The transportation model (between warehouse) is balanced (sum of the supply = sum of the demand).
3. For each warehouse has its own delivery area (independent delivery area) and becomes the root vertex in its own delivery area.

Therefore, the three assumptions to be able to use the combination of VAM and Dijkstra algorithm are fulfilled.

Table 4. Distribution Cost Between Warehouse (Based on Supply and Demand)

Warehouse	G_3	G_4
G_1	5	7
G_2	8	4

Based on the table above, the minimum distribution costs using the VAM are determined.

Table 5. The Iteration of VAM

Iteration 1	G_3	G_4	Supply	Penalty	Iteration 2	G_3	G_4	Supply	Penalty
G_1	5	7	15	2	G_1	5	7	15	2
G_2	8	4	5	4	G_2	-	4{5}	5-5=0	
Demand	10	10			Demand	10	10-5=5		
Penalty	3	3			Penalty	5	7		
Iteration 3	G_3	G_4	Supply	Penalty	Iteration 4	G_3	G_4	Supply	Penalty
G_1	5	7{5}	15-5=10	5	G_1	5{10}	7{5}	10-10=0	
G_2	-	4{5}	0		G_2	-	4{5}	0	
Demand	10	5-5=0			Demand	10-10=0	0		
Penalty	5				Penalty				

Based on the tabel above, the minimum distribution costs graph is obtained.

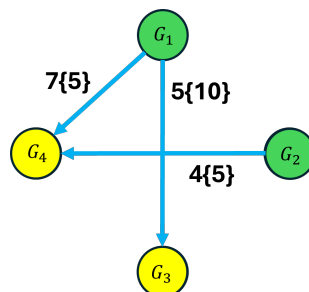


Figure 3. VAM for Warehouses

The minimum distribution costs between warehouses are obtained as follows:

1. warehouse G_1 distributes 5 to warehouse G_3 and 7 to warehouse G_4 ;

2. warehouse G_2 distributes 5 to warehouse G_4

with the minimum distribution costs between warehouses

$$Cw = (5 \times 10) + (7 \times 5) + (4 \times 5) = 105.$$

Thus, there are no warehouses with inventory shortages.

Next, determine the shortest distribution route from the warehouse to each agent using Dijkstra algorithm. The following is the shortest distribution route from warehouse G_3 to each agent.

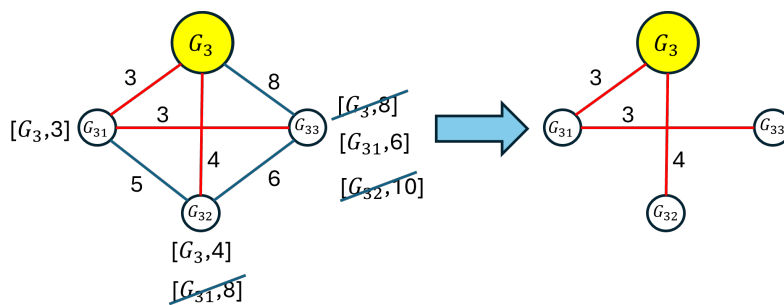


Figure 4. Dijkstra Algorithm for Warehouse G_3 and Its Agents

For other warehouses, use the same method. There for, distribution route between warehouse and its agents is obtained.

Table 6. Distribution Route Between Warehouse and Its Agents

Initial Warehouse	Destination Agent	Route
G_1	G_{11}	$G_1 \rightarrow G_{11}$
G_1	G_{12}	$G_1 \rightarrow G_{11} \rightarrow G_{12}$
G_2	G_{21}	$G_2 \rightarrow G_{21}$
G_2	G_{22}	$G_2 \rightarrow G_{25} \rightarrow G_{22}$
G_2	G_{23}	$G_2 \rightarrow G_{21} \rightarrow G_{23}$
G_2	G_{24}	$G_2 \rightarrow G_{21} \rightarrow G_{24}$
G_2	G_{25}	$G_2 \rightarrow G_{25}$
G_3	G_{31}	$G_3 \rightarrow G_{31}$
G_3	G_{32}	$G_3 \rightarrow G_{32}$
G_3	G_{33}	$G_3 \rightarrow G_{31} \rightarrow G_{33}$
G_4	G_{41}	$G_4 \rightarrow G_{41}$
G_4	G_{42}	$G_4 \rightarrow G_{44} \rightarrow G_{42}$
G_4	G_{43}	$G_4 \rightarrow G_{44} \rightarrow G_{43}$
G_4	G_{44}	$G_4 \rightarrow G_{44}$

Finally, the following is a problem solution graph obtained by combining the VAM and

Dijkstra algorithm.

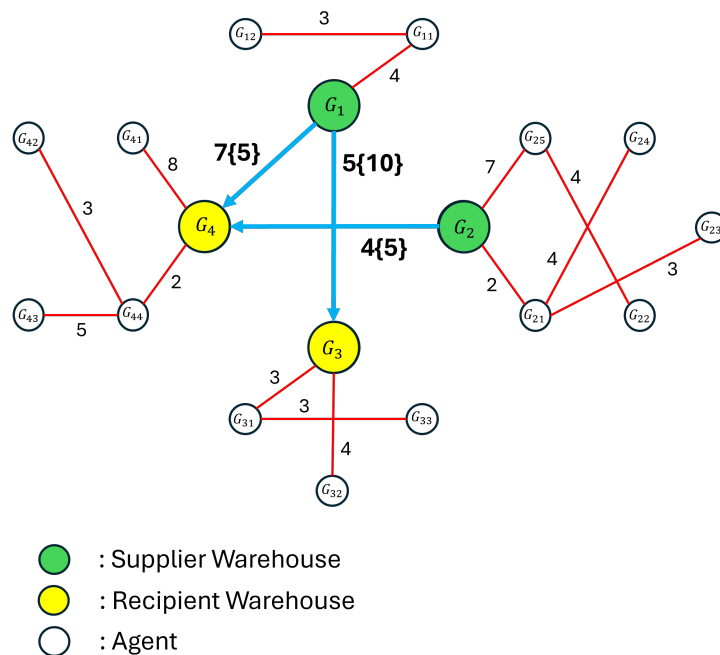


Figure 5. Distribution Graph

The minimum distribution cost from G_i to each G_{in_i} , for $i = 1, 2, 3, 4$ is

$$\begin{aligned} Cw_1 &= 4 + 3 = 7, \\ Cw_2 &= 2 + 7 + 4 + 4 + 3 = 20, \\ Cw_3 &= 3 + 3 + 4 = 10, \\ Cw_4 &= 8 + 2 + 3 + 5 = 18. \end{aligned}$$

And, the total distribution cost is $C = Cw + \sum_{i=1}^4 Cw_i = 105 + (7 + 20 + 10 + 18) = 160\$$.

IV. CONCLUSIONS AND FUTURE RESEARCH DIRECTION

Vogel's Approximation Method (VAM) and Dijkstra's algorithm can be combined if three assumptions are met. These three assumptions are based on the characteristics of each VAM and Dijkstra's algorithm, as well as the compatibility between the two. In simple terms, the three assumptions are: 1) there is at least one warehouse that has excess capacity and one warehouse that lacks capacity; 2) the transportation model (between warehouse) is always balanced; and 3) for each warehouse has its own delivery area (independent delivery area).

REFERENCES

- [1] D. G. Luenberger, and Ye, Y., *Linear and Nonlinear Programming Third Edition*. New York: Springer Science+Business Media, 2008.
- [2] G. B. Dantzig and M. N. Thapa, *Linear Programming 1: Introduction*. New York: Springer-Verlag New York, 1997.

- [3] H. A. Taha, *Operations Research: An Introduction Tenth Edition*. Pearson, 2017.
- [4] S. R. Wahyu, A. Rohima, K. F. Handayani, and M. Fauzi, "Optimalisasi Biaya Distribusi Kain Mentah di PT PQR Menggunakan Metode VAM (Vogel's Approximation Method) dan LINGO," *Jurnal Bayesian : Jurnal Ilmiah Statistika dan Ekonometrika*, vol. 2, no. 1, pp. 91–99, 2021.
- [5] M. Kempa, "Implementasi Metode Vogel's Approximation Method (VAM) dan Stepping Stone untuk Optimalisasi Biaya Distribusi Material Besi Beton pada Daerah Kepulauan di Provinsi Maluku," *Jurnal SIMETRIK*, vol. 12, no. 1, pp. 504–511, 2022.
- [6] S. Sapna and Y. Iriani, "Penerapan Metode North West Corner, Least Cost, dan Vogel Approximation untuk mengoptimalkan Biaya Transportasi pada Distribusi Panel Surya Ke Setiap lokasi Proyek di PT XYZ," *Lebesgue: Jurnal Ilmiah Pendidikan Matematika, Matematika dan Statistika*, vol. 6, no. 1, pp. 266–277, 2025.
- [7] N. Pratiwi and R. Siregar, "Optimization of Crude Palm Oil Distribution Costs in PT. Perkebunan Nusantara III Using Vogel's Approximation Method, Russel Approximation Method and Stepping Stone Method," *Journal of Mathematics Technology and Education*, vol. 1, no. 1, pp. 1–10, 2021.
- [8] W. Arimurti, R. P. Sari, D. Herwanto, and C. Falah, "Optimasi Biaya Transportasi Pengiriman Produk Mainan Menggunakan Vogel's Approximation Method Dan Stepping Stone Method (Studi Kasus: Toko Sumber Mainan)," *SITEKIN: Jurnal Sains, Teknologi dan Industri*, vol. 20, no. 1, pp. 365–374, 2022.
- [9] Y. Ratnasari, D. Yuniarti, and I. Purnamasari, "Optimasi Pendistribusian Barang Dengan Menggunakan Vogel's Approximation Method dan Stepping Stone Method (Studi Kasus: Pendistribusian Tabung Gas LPG 3 Kg Pada PT. Tri Pribumi Sejati)," *Jurnal EKSPONENSIAL*, vol. 10, no. 2, pp. 165–174, 2019.
- [10] N. Hermanto, E. H. Hermaliani, and E. Sutinah, "Optimasi Pendistribusian Barang Dengan Menggunakan Vogel's Approximation Method dan Stepping Stone Method (Studi Kasus: Pendistribusian Tabung Gas LPG 3 Kg Pada PT. Tri Pribumi Sejati)," *Jurnal Teknik Komputer AMIK BSI*, vol. 3, no. 1, pp. 30–36, 2017.
- [11] I. Arifin, S. Rahmansyah, S. N. Fauziyyah, and M. Fauzi, "Minimasi Biaya Pengiriman Tahu menggunakan Metode Transportasi," *Jurnal Taguchi: Jurnal Ilmiah Teknik dan Manajemen Industri*, vol. 2, no. 1, pp. 37–45, 2022.
- [12] N. L. Azizah and M. Suryawinata, "Aplikasi Metode Transportasi Dalam Optimalisasi Biaya Distribusi Beras Sejahtera Pada PERUM BULOG Sub-Divre Sidoarjo," *Jurnal Ilmiah: SOULMATH*, vol. 6, no. 1, pp. 15–23, 2018.
- [13] A. B. H. Yanto, "Penerapan Metode VAM Dalam Optimalisasi Biaya Pengiriman Spare Part pesawat Pada PT. Aviastar Mandiri," *Jurnal Teknologi Informatika dan Komputer*, vol. 5, no. 1, pp. 36–44, 2019.
- [14] N. Dimasuharto, A. M. Subagyo, and R. Fitriani, "Optimalisasi Biaya Pendistribusian Produk Kaca Menggunakan Model Transportasi dan Metode Stepping Stone" *INTECH*, vol. 7, no. 2, pp. 81–88, 2021.

- [15] N. M. A. Pranati, A. I. Jaya, and A. Sahari, "Optimasi Biaya transportasi pendistribusian Keramik Menggunakan Model Transtortasi Metode Stepping Stone (Studi kasus: PT. Indah Bangunan)" *JIMT: Jurnal Ilmiah Matematika dan Terapan*, vol. 15, no. 1, pp. 48–57, 2018.
- [16] M. C. Bunaen, H. Pratiwi, and Y. F. Riti, "Penerapan Algoritma Dijkstra Untuk Menentukan Rute Terdekat Dari Pusat Kota Surabaya ke Tempat Bersejarah," *Jurnal Teknologi Dan Sistem Informasi Bisnis*, vol. 4, no. 1, pp. 213–223, 2022.
- [17] M. Nandiroh, Haryanto, and Y. Munawir, "Implementasi Algoritma Dijkstra Sebagai Solusi Efektif Pembuatan Sistem Bantuan Bencana Real Time," *Jurnal Ilmiah Teknik Industri*, vol. 12, no. 2, pp. 223–234, 2013.
- [18] M. Qomaruddin, M. T. Alawy, and S. Sugiono, "Perancangan Aplikasi Penentu Rute Terpendek Perjalanan Wisata di Kabupaten Jember Menggunakan Algoritma Dijkstra," *Sci. Electro*, vol. 6, no. 2, pp. 31–39, 2018.
- [19] D. Wahyuningsih and E. Syahreza, "Shortest Path Search Futsal Field Location with Dijkstra Algorithm," *IJCCS (Indonesian J. Comput. Cybern. Syst.*, vol. 12, no. 2, pp. 161–170, 2018.
- [20] A. Amin and B. Hendrik, "Analisis Penerapan Algoritma Dijkstra dalam Optimasi Penentuan Rute: Sebuah Kajian Literatur Sistematis," *Journal of Education Research*, vol. 6, no. 1, pp. 100–106, 2025.
- [21] M. Wiladi, Wasono, and Asmaidi, "Penerapan Algoritma Dijkstra dan Algoritma Greedy Pada Optimasi Jalur Evakuasi Banjir," *Basis: Jurnal Ilmiah Matematika*, vol. 2, no. 1, pp. 25–38, 2023.
- [22] M. Andini, R. U. Kultsum, M. H. R. Raihan and S. Lestari, "Optimasi Pencarian Rute Terpendek Menggunakan Algoritma Dijkstra," *Jisamar*, vol. 9, no. 1, pp. 290–302, 2025.
- [23] R. Wahyudi, M. Alfin, J. B. Henrydunan, and P. Harliana, "Penerapan Algoritma Dijkstra untuk Optimasi Rute Terpendek dari Fakultas Kedokteran UNIMED ke Empat Gerbang Kampus Menggunakan Python," *JATI : Jurnal Mahasiswa Teknik Informatika*, vol. 8, no. 6, pp. 12073–12078, 2024.
- [24] K. Hermanto, and T. D. Ermayanti, "Analisa Optimasi Rute Transportasi Antar Jemput Siswa Menggunakan Model CGVRP dan Algoritma Dijkstra di SDIT Darus Sunnah," *UJMC (Unisda Journal of Mathematics and Computer Science)*, vol. 5, no. 2, pp. 19–28, 2019.
- [25] N. F. Lakutu, M. R. Katili, S. L. Mahmud, and N. I. Yahya, "Algoritma Dijkstra dan Algoritma Greedy Untuk Optimasi Rute Pengiriman Barang Pada Kantor Pos Gorontalo," *EULER: Jurnal Ilmiah Matematika, Sains dan Teknologi*, vol. 11, no. 1, pp. 55–65, 2023.