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SOMBOR INDEX AND ITS GENERALIZATION OF POWER GRAPH OF SOME GROUP WITH PRIME POWER ORDER

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Abstract. Graphs are an intriguing topic of discussion due to their numerous applications, particularly in chemistry. Topological indices derived from graph representations of molecules enable us to predict various properties of these compounds, including their physical characteristics, chemical reactivity, biological activity, toxicity, and atom-to-atom interactions. More recently, graphs have also been utilized to depict abstract mathematical objects such as groups. A notable example of graph representation in group theory is seen in power graphs. This research explores new graph topological indices based on vertex degrees, inspired by the Euclidean metric, particularly the Sombor index, and its application to the power graph of the integer modulo group and the dihedral group. The primary outcome of this study is the derivation of a general formula for the Sombor index and its generalization.

Keywords: Power Graph, Sombor Index, Dihedral Group, Integer Modulo.

I. INTRODUCTION

A graph is a mathematical concept that describes relationships between objects within a system, while a group is an abstract mathematical object. The intersection of these two areas is particularly intriguing to explore, with topics such as the coprime graph [1] and the non-coprime graph [2] representing popular methods of graphically depicting group elements based on their orders. Other representation of a group using graphs is the power graph, where the elements of the group is adjacent when one is equal to the power of other [3]. Graph theory and group theory are concepts related to topological indexes.

Graphs have applications beyond mathematics; they are also essential in the field of chemistry. Topological indices play a crucial role in analyzing the physical characteristics, chemical reactivity, and biological activity of chemical compounds [4]. These indices provide insights into a compound's biological activity, toxicity, and atom-to-atom interactions [5]. Many mathematicians study various topological indices, including well-known measures such as the Wiener index, Zagreb index, Gutman index, Harmonic index, Szeged index, and Padmakar-Ivan index. These indices play a vital role in graph theory and molecular graph theory, giving valuable help into the structural properties and characteristics of molecules and networks. The Wiener index, for instance, quantifies the total distance between every two vertices in a graph, reflecting molecular size and branching [6]. The Zagreb index measures vertex connectivity by summing the squares of vertex degrees [7], while the Gutman index considers eigenvalues of molecular graphs. The Harmonic index assesses graph complexity based on vertex degrees and distances [8], and the Szeged index accounts for edge-vertex contributions in a molecular structure [9]. The Padmakar-Ivan index, a more recent addition, evaluates molecular complexity and stability using a unique vertex-edge metric [10]. These indices contribute significantly to the understanding of chemical compounds and their properties, aiding in drug design, chemical synthesis, and network analysis.

In this research, we introduces a new graph indices based on vertex degree, called the Sombor index, Reduced Sombor index, and Average Sombor index [11]. The Sombor index is defined as the sum of squared values of vertex degrees in the graph, aiming to quantify the relationships between atoms within a molecule based on their degree values. The Reduced Sombor index adjusts the calculation by excluding degrees smaller than one, thereby mitigating the influence of lower-degree vertices that may distort the Sombor index value [12]. Similarly, the Average Sombor index modifies the sum of squared degree values by reducing each vertex's degree based on the total number of edges (m) and vertices (n), incorporating a normalization factor of $\frac{2m}{n}$ [11].

II. RESULTS

Some definitions of power graph, integer group modulo and dihedral group. which are used in this research are as follows.

Definition 1 [13] The p*ower graph of a group H is defined as a graph whose vertices set is all element of group H and two diferent vertices v_1 and v_2 are ajacent if only if ${v_1}^x = v_2$ or $v_2^y = v_1$ for a natural number x and y.*

Definition 2 [14] *A group D_{2n} is called a dihedral group with $n \geq 3$ and $n \in \mathbb{N}$, is a group generated by two element $x, y \in D_{2n}$ which is denoted by $D_{2n} = \langle x, y | x^n = e, y^2 = e, yxy^{-1} = x^{-1} \rangle$.*

Definition 3 [15]*Group $\mathbb{Z}_n$ is called integer group modulo n, with its members being positive integers with addition operation modulo n. Denoted by $\mathbb{Z}_n = \{0, 1, 2, \dots, n-1\}$ with operation $+_{mod(n)}$.*

To get the power graph of the group, we need to know the neighboring vertices. here's an example power graph on the group of integers modulo *n* and the dihedral group with *n* a prime power.

Example 1. Suppose given $\mathbb{Z}_3 = \{0, 1, 2\}$.

Table 1. Table Neighbord in $\mathbb{Z}_3$

Element $\mathbb{Z}_3$	**0**	**1**	**2**
0	-	$1^3 = 0$	$2^3 = 0$
1	$1^3 = 0$	-	$1^2 = 2$
2	$2^3 = 0$	$1^2 = 2$	-

So that the graph image obtained from the adjacency shown in Table 1 is obtained as illustrated in Fig. 1.

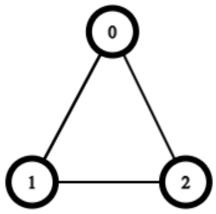

Figure 1. The image for power graph of $\mathbb{Z}_3$

Example 2. Suppose given the group $D_6 = \{e, a, a^2, b, ab, a^2b\}$.

Table 2. Table Neighbord in D_6

$\boldsymbol{Element\ \ D_{23}}$	**e**	**a**	$\boldsymbol{a^2}$	**b**	$\boldsymbol{ab}$	$\boldsymbol{a^2b}$
e	-	$a^3 = e$	$(a^2)^3 = e$	$b^2 = e$	$(ab)^2 = e$	$(a^2b)^2 = e$
a	$a^3 = e$	-	$(a)^2 = a^2$	-	-	-
$\boldsymbol{a^2}$	$(a^2)^3 = e$	$(a)^2 = a^2$	-	-	-	-
b	$b^2 = e$	-	-	-	-	-
ab	$(ab)^2 = e$	-	-	-	-	-
$\boldsymbol{a^2b}$	$(a^2b)^2 = e$	-	-	-	-	-

So that the graph image obtained from the adjacency shown in Table 2 is obtained as illustrated in Fig. 2.

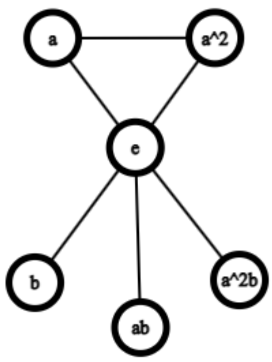

Figure 2. The image for power graph of D_6

From these examples we can determine the pattern of the degree of each vertex and the structure of the power graph of the group. Definition of degree is given bellow.

Definition 4 [16] *Degree of a vertex in a simple graph is the number of edges adjacent to the vertex, it is denoted as $deg(a)$ for an arbitrary vertex a.*

In 2023, Putra, et al and in 2021, Asmarani, et al. Successively found the form of the power graph of the integer group and the dihedral group.

Theorem 1 [12] *If $\mathbb{Z}_n$ is integer modulo with $n = q^k$, where $k \in \mathbb{N}$, and q prime number, then the power graph of the integer modulo group $\mathbb{Z}_n$, denoted by $\Gamma_{\mathbb{Z}_n}$, is form a complete graph (K_n).*

Theorem 2 [14] *If D_{2n} with $n = p^k$ where $k \in \mathbb{N}$ and p prime number, then the power graph of the dihedral group has two subgraphs which is a complete graph and a star graph.*

Based on **Theorem 1**, the degree of the vertices of the vertices in the group of integers modulo.

Theorem 3 [12] *Degree of vertices of the power graph of the group of integers modulo $n = p^k$, p prime, is $deg\ (a) = n - 1\ \forall a \in \mathbb{Z}_n$*

Based on **Theorem 2**, the degree of the vertices of the vertices in the dihedral group.

Theorem 4 [14] *Degree of vertices of power graph of dihedral group D_{2n}, $n = p^k$, p prime is*

a. *deg(e)*= $2n - 1$
b. $deg\ (a^i b) = 1\ \ \forall i \in \mathbb{Z}\ , \mathbb{Z}_n 0 \le i \le n - 1$
c. $deg\ (a^j) = n - 1\ \ \forall j \in \mathbb{Z}\ , \mathbb{Z}_n 1 \le j \le n - 1$

Recently discovered vertex degree-based topological index is the sombor index and its generalization, defined as follows.

Definition 5 [17] *Suppose G simple connected graph with $V(G)$ is the set of vertices, and $E(G)$ is the set of edges. Sombor index, reduced sombor index, average sombor index (where m is the number of edges and n is the number of vertices) of graf G are defined the following:*

$$SO(G) = \sum_{\{uv\} \in E(G)} \sqrt{deg\ (u)^2 + deg\ (v)^2}$$

$$SO_{red}(G) = \sum_{\{uv\} \in E(G)} \sqrt{(deg(u) - 1)^2 + (deg(v) - 1)^2}$$

$$SO_{avg}(G) = \sum_{\{uv\} \in E(G)} \sqrt{(deg(u) - \frac{2m}{n})^2 + (deg(v) - \frac{2m}{n})^2}$$

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so that the sombor index and its generalization of the group of integers modulo are as follows.

Theorem 5 *Given an integer group modulo n ($\mathbb{Z}_n$) with operation $+_{mod(n)}$. With n being a prime power number* $SO\left(\Gamma_{\mathbb{Z}_n}\right) = \frac{\sqrt{2}n(n-1)^2}{2}$.

Proof: Given an integer group modulo n ($\Gamma_{\mathbb{Z}_n}$), based **Theorem 3,** $\deg(a) = n - 1, \forall a \in V(G)$, then

$$SO\left(\Gamma_{\mathbb{Z}_n}\right) = \sum_{\{uv\}\in E\left(\Gamma_{\mathbb{Z}_n}\right)} \sqrt{\deg(u)^2 + \deg(v)^2}$$
$$=\sqrt{(n-1)^2 + (n-1)^2} + \sqrt{(n-1)^2 + (n-1)^2} + \cdots + \sqrt{(n-1)^2 + (n-1)^2}$$
$$=\sqrt{(n-1)^2 + (n-1)^2} + \sqrt{(n-1)^2 + (n-1)^2} + \cdots + \sqrt{(n-1)^2 + (n-1)^2}$$

based on **Theorem 1** multiple edges $\binom{n}{2}$

$$= \binom{n}{2}\sqrt{(n-1)^2 + (n-1)^2}$$
$$= \frac{n(n-1)(n-2)!}{(n-2)!2!} \sqrt{2(n-1)^2}$$
$$= \frac{n(n-1)}{2} (n-1)\sqrt{2}$$
$$= \frac{n(n-1)^2\sqrt{2}}{2} . \blacksquare$$

Theorem 6 *Given an integer group modulo n ($\mathbb{Z}_n$) with operation $+_{mod(n)}$. With n being a prime power number, then reduced sombor index* $SO_{red}(\Gamma_{\mathbb{Z}_n}) = \frac{(n^2-n)(n-2)\sqrt{2}}{2}$.

Proof: *Ba*sed on **Theorem 3** we get, $\deg(a) = n - 1, \forall a \in V(G)$

$$SO_{red}\left(\Gamma_{\mathbb{Z}_n}\right) = \sum_{\{uv\}\in E(\Gamma_{\mathbb{Z}_n})} \sqrt{(\deg(u) - 1)^2 + (\deg(v) - 1)^2}$$
$$=\sqrt{((n-1)-1)^2 + ((n-1)-1)^2} + \cdots + \sqrt{((n-1)-1)^2 + ((n-1)-1)^2}$$

based on **Theorem 1** multiple edges $\binom{n}{2}$

$$= \binom{n}{2}\sqrt{2((n-1)-1)^2 + ((n-1)-1)^2}$$
$$= \frac{n(n-1)\sqrt{2(n-2)^2}}{2}$$
$$= \frac{(n^2-n)(n-2)\sqrt{2}}{2}. \blacksquare$$

Theorem 7 *Given an integer group modulo n ($\mathbb{Z}_n$) with operation $+_{mod(n)}$. With n being a prime power number, then average sombor index* $SO_{avg}\left(\Gamma_{\mathbb{Z}_n}\right) = 0$.

Proof : Based on **Theorem 3,** $\deg(a) = n - 1, \forall a \in V(G)$, and m is the number of edges and n is the number of vertices

$$SO_{avg}(\Gamma_{\mathbb{Z}_n}) = \sum_{\{uv\}\in E(\Gamma_{\mathbb{Z}_n})} \sqrt{(\deg(u) - \frac{2m}{n})^2 + (\deg(v) - \frac{2m}{n})^2}$$

$$= \sqrt{((n-1) - \frac{2m}{n})^2 + ((n-1) - \frac{2m}{n})^2} + \cdots + \sqrt{((n-1) - \frac{2m}{n})^2 + ((n-1) - \frac{2m}{n})^2}$$

Based **Theorem 1** multiple edges $\binom{n}{2}$

$$= \binom{n}{2}\sqrt{((n-1) - \frac{2\binom{n}{2}}{n})^2 + ((n-1) - \frac{2\binom{n}{2}}{n})^2}$$
$$=\frac{n(n-1)}{2}\sqrt{((n-1) - \frac{2n(n-1)}{2n})^2 + ((n-1) - \frac{2n(n-1)}{2n})^2}$$
$$=\frac{n(n-1)}{2}\sqrt{((n-1) - (n-1))^2 + ((n-1) - (n-1))^2}$$
$$=\frac{n(n-1)}{2}\sqrt{(0)^2 + (0)^2}$$
$$= 0.$$

Similar to the Sombor index and its generalization on integers modulo n, we can formulate the Sombor index and its generality in the dihedral group, by inserting some degree of a known vertex and multiplying by the number of edges of the vertex.

Theorem 8 *Given dihedral grup* (D_{2n})*. With* n *being a prime power number, then sombor index* $SO(\Gamma_{D_{2n}}) = (n-1)\sqrt{5n^2 - 6n + 2} + \frac{\sqrt{2}}{2}(n-1)^2(n-2) + n\sqrt{4n^2 - 4n + 2}$.

Proof : Based on **Theorem 2,** the power graph of the dihedral group forms two subgraphs. Based on **Theorem 4,** $\deg(e) = 2n - 1, \deg(a^i b) = 1 \ \forall i \in \mathbb{Z}, \ 0 \le i \le n-1$, $\deg(a^j) = n - 1 \ \forall j \in \mathbb{Z} \ 1 \le j \le n-1$. Therefore, it is divided into three cases:

a) when e neighbors a^j

$$SO(\Gamma_{D_{2n}}) = \sum_{\{ea^j\}\in E(\Gamma_{D_{2n}})} \sqrt{\deg(e)^2 + \deg(a^j)^2}$$

e neighbors every a^j, multiple edges $\{ea^j\} \in E(D_{2n}) = n - 1$

$$=(n-1)\sqrt{(2n-1)^2 + (n-1)^2}$$
$$=(n-1)\sqrt{(4n^2 - 4n + 1) + (n^2 - 2n + 1)}$$
$$=(n-1)\sqrt{5n^2 - 6n + 2}$$

b) when a neighbors a^{j-1}, $3 \le j \le n$

$$SO(\Gamma_{D_{2n}}) = \sum_{\{aa^{j-1}\}\in E(\Gamma_{D_{2n}})} \sqrt{\deg(a)^2 + \deg(a^{j-1})^2}$$

multiple edges $\{aa^{i-1}\} \in E(D_{2n}) = \binom{n-1}{2}$

$$= \binom{n-1}{2}\sqrt{(n-1)^2 + (n-1)^2}$$

$$= \frac{(n-1)(n-2)}{2}\sqrt{2(n-1)^2}$$

$$= \frac{(n-1)^2(n-2)\sqrt{2}}{2}$$

c) e neighbors a^ib, $0 \le i \le n-1$

$$SO(\Gamma_{D_{2n}}) = \sum_{\{ea^ib\}\in E(\Gamma_{D_{2n}})} \sqrt{\deg(e)^2 + \deg(a^ib)^2}$$

multiple edges is n

$$= n\sqrt{(2n-1)^2 + 1^2}$$

$$= n\sqrt{4n^2 - 4n + 2}$$

Then

$$SO(\Gamma_{D_{2n}}) = (n-1)\sqrt{5n^2 - 6n + 2} + \frac{(n-1)^2(n-2)\sqrt{2}}{2} + n\sqrt{4n^2 - 4n + 2}. \blacksquare$$

Theorem 9 *Given dihedral grup (D_{2n}). With n being a prime power number, then reduced sombor index* $SO_{red}(\Gamma_{D_{2n}}) = (n-1)\sqrt{5n^2 - 8n + 8} + \frac{(n-1)(n-2)^2\sqrt{2}}{2} + n(2n-2)$.

Proof : same as **Theorem 8** divided into three cases:

a) when e neighbors a^j

$$SO_{red}(\Gamma_{D_{2n}}) = \sum_{\{ea^j\}\in E(\Gamma_{D_{2n}})} \sqrt{(\deg(e)-1)^2 + (\deg(a^j)-1)^2}$$

$$=(n-1)\sqrt{((2n-1)-1)^2 + ((n-1)-1)^2}$$

$$=(n-1)\sqrt{(2n-2)^2 + (n-2)^2}$$

$$=(n-1)\sqrt{4n^2 - 4n + 4 + n^2 - 4n + 4}$$

$$=(n-1)\sqrt{5n^2 - 8n + 8}$$

b) a neigbhors a^{j-1}, $3 \le j \le n$

$$SO_{red}(\Gamma_{D_{2n}}) = \sum_{\{aa^{j-1}\}\in E(\Gamma_{D_{2n}})} \sqrt{(\deg(a)-1)^2 + (\deg(a^{j-1})-1)^2}$$

$$= \binom{n-1}{2}\sqrt{((n-1)-1)^2 + ((n-1)-1)^2}$$

$$= \frac{(n-1)(n-2)}{2}\sqrt{(n-2)^2 + (n-2)^2}$$

$$= \frac{(n-1)(n-2)}{2}\sqrt{2(n-2)^2}$$

$$= \frac{(n-1)(n-2)(n-2)\sqrt{2}}{2}$$

$$= \frac{(n-1)(n-2)^2\sqrt{2}}{2}$$

c) e neighbors a^ib, $0 \le i \le n-1$

$$SO_{red}(\Gamma_{D_{2n}}) = \sum_{\{ea^{i-1}\}\in E(\Gamma_{D_{2n}})} \sqrt{(\deg(e)-1)^2 + (\deg(a^{i-1})-1)^2}$$

$$= n\sqrt{\left((2n-1)-1\right)^2 + (1-1)^2}$$

$$= n\sqrt{(2n-2)^2}$$

$$= n(2n-2)$$

Then

$$SO_{red}(\Gamma_{D_{2n}}) = (n-1)\sqrt{5n^2-8n+8} + \frac{(n-1)(n-2)^2\sqrt{2}}{2} + n(2n-2). \blacksquare$$

Theorem 10 *Given dihedral grup (D_{2n}). With n being a prime power number, then average sombor index* $SO_{avg}(\Gamma_{D_{2n}}) = \frac{n-1}{n}\sqrt{(2n^2-3n+2)^2+(n^2-3n+1)^2} + \frac{(n-1)(n-2)(2n-2)\sqrt{2}}{2n} + n\sqrt{4n^2-12n+10}$.

Proof : Like the theorems before, we divided into three cases, with m being the number of edges and n the number of vertices.

a) e neighbors a^j

$$SO_{avg}(\Gamma_{D_{2n}}) = \sum_{\{ea^j\}\in E(\Gamma_{D_{2n}})} \sqrt{(\deg(e)-\frac{2m}{n})^2 + (\deg(a^j)-\frac{2m}{n})^2}$$

$$= (n-1)\sqrt{((2n-1)-\frac{2(n-1)}{n})^2 + ((n-1)-\frac{2(n-1)}{n})^2}$$

$$= (n-1)\sqrt{(\frac{2n^2-3n+2}{n})^2 + (\frac{n^2-3n+1}{n})^2}$$

$$= (n-1)\sqrt{\frac{1}{n^2}(2n^2-3n+2)^2 + (n^2-3n+1)^2}$$

$$= \frac{n-1}{n}\sqrt{(2n^2-3n+2)^2 + (n^2-3n+1)^2}$$

b) a neigbhors a^{j-1}, $3 \le j \le n$

$$SO_{avg}(\Gamma_{D_{2n}}) = \sum_{\{aa^{j-1}\}\in E(\Gamma_{D_{2n}})} \sqrt{(\deg(a) - \frac{2m}{n})^2 + (\deg(a^{j-1}) - \frac{2m}{n})^2}$$

$$= \binom{n-1}{2}\sqrt{((n-1) - \frac{2\binom{n-1}{2}}{n})^2 + ((n-1) - \frac{2\binom{n-1}{2}}{n})^2}$$

$$= \frac{(n-1)(n-2)}{2}\sqrt{2((n-1) - \frac{2\binom{n-1}{2}}{n})^2}$$

$$= \frac{(n-1)(n-2)}{2}\sqrt{2((n-1) - \frac{2(n-1)(n-2)}{2n})^2}$$

$$= \frac{(n-1)(n-2)}{2}\sqrt{2((n-1) - \frac{(n-1)(n-2)}{n})^2}$$

$$= \frac{(n-1)(n-2)}{2}\sqrt{2(\frac{(2n-2)}{n})^2}$$

$$= \frac{(n-1)(n-2)(2n-2)\sqrt{2}}{2n}$$

c) e neighbors $a^i b$, $0 \le i \le n-1$

$$SO_{avg}(\Gamma_{D_{2n}}) = \sum_{\{ea^i b\}\in E(\Gamma_{D_{2n}})} \sqrt{(\deg(e) - \frac{2m}{n})^2 + (\deg(a^i b) - \frac{2m}{n})^2}$$

$$= n\sqrt{((2n-1) - \frac{2n}{n})^2 + (1 - \frac{2n}{n})^2}$$

$$= n\sqrt{(2n-3))^2 + (1-2)^2}$$

$$= n\sqrt{4n^2 - 12n + 10}$$

Then

$$SO_{avg}(\Gamma_{D_{2n}}) = \frac{n-1}{n}\sqrt{(2n^2-3n+2)^2 + (n^2-3n+1)^2} + \frac{(n-1)(n-2)(2n-2)\sqrt{2}}{2n} + n\sqrt{4n^2-12n+10}\,. \blacksquare$$

III. CONCLUSIONS AND FUTURE RESEARCH DIRECTION

In this study, we derive the general formulas for calculating the Sombor index, reduced Sombor index, and average Sombor index of the power graph associated with the integer group $\mathbb{Z}_n$ and the dihedral group D_{2n}, where n is a prime power number. The Sombor index and its corresponding generalizations for the integer group modulo n are rigorously established and detailed in Theorems 5, 6, and 7. Similarly, the results for the dihedral group are provided in Theorems 8, 9, and 10. These findings contribute to a comprehensive understanding of the structural properties of these mathematical structure, shedding light on their graph-theoretic characteristics.

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