JOURNAL OF FUNDAMENTAL MATHEMATICS
AND APPLICATIONS (JFMA) VOL. 7 NO. 1 (JUN 2024)
Available online at www.jfma.math.fsm.undip.ac.id

ANALYSIS OF THE EFFECT OF STUART NUMBER AND RADIATION ON VISCOUS FLUID FLOW

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Abstract. Computational fluid dynamics (CFD) is a numerical solution of fluid flow problems built from applied mathematical modeling. The problem of the flow of a viscous fluid which is influenced by a magnetic field gives rise to a boundary layer, from this boundary layer a dimensional building equation is formed. The governing equation is obtained from the continuity equation, momentum equation, and energy equation, then transformed into a non-dimensional equation by substituting non-dimensional variables and transformed into a similarity equation. The numerical solution to this problem uses the Keller Box method. The numerical simulation results show that the Stuart Number increases the velocity profile, while the temperature profile decreases. The effect of radiation parameters on the velocity profile did not change significantly, but the temperature profile decreased.
Keywords: CFD, Radiation Parameter, Stuart Number, Viscous Fluid.

I. INTRODUCTION

Regardless of the amount of shear force applied, fluids are substances that constantly change shape when subjected to shear stress [1]. Gases and liquid-like compounds are considered fluids. Newtonian fluids and non-Newtonian fluids are the two categories into which liquids can be classified. A non-Newtonian fluid changes its viscosity in response to external forces, whereas a Newtonian fluid remains constant in viscosity despite external forces acting on it. Fluids have unique features due to their nature, including density (mass), viscosity (thickness), and compressibility (compressibility).

A fluid that is affected by viscosity (thickness) is called viscous. The forces created when a layer of fluid scrapes against another item are what cause viscosity. Any flow that takes place in a viscous fluid is called viscous flow. The friction between the fluid's constituent particles determines the viscosity of viscous flow. Numerous researchers continue to advance this discipline, with viscous fluid being one area that is impacted by MHD [2]. Furthermore, blood flow can be used to illustrate viscous flow. In this case, it is thought that venous blood flows in the direction of the heart and onto the lungs of isolated cats. Viscous flow plays a significant part in health-related research since it has been used to study stenotic blood flow, which produces blockages in the blood. [3], [4].

Heat transfer is a science that studies the rate of heat transfer between objects due to temperature differences [5], [6], [7]. There are three possible methods for heat transmission to happen: convection, radiation, and conductor heat transfer. The transfer of heat that happens when electromagnetic waves are emitted without the use of a medium in between is called radiation. In contrast to conduction and convection heat transfer, which depend on the presence of a heat transfer medium, radiation events will transpire more successfully in a vacuum.

The Keller-Box method is used to solve partial differential equations by changing high order to first order [5], [8]. Fluid flow problems are often solved using the Keller-Box method to see the characteristics of the flow [9]. The heat transmission properties of viscoelastic fluids in the presence of nanoparticles were investigated theoretically. In addition, the energy equation contains a mathematical description of viscous dissipation and is then solved using the Keller box method [10]. In addition, numerical issues involving changes in mass and heat can be resolved using the Keller-Box method [11].

A fundamental investigation into the motion of viscous fluid flow will be done in this work. We will look at the effects of radiation and magnetohydrodynamics (MHD) in this last assignment. This study examines the fundamental relationship between radiation and magnetohydrodynamics (MHD) in fluid movement. The problem being studied is described as follows: a boundary layer is created when flow passes through a circular cylinder. The fluid flow in this problem moves from above and then passes through a circular cylinder with radius a which is immersed in an incompressible viscous fluid and the surrounding temperature is T_∞ and the temperature on the cylinder wall is T_w. The $\bar{x}$ coordinate is the line along the surface of the circular cylinder, the $\bar{y}$ coordinate is the normal line to the surface of the circular cylinder. The boundary layer equation must be formulated and solved before the dimensional boundary layer equation can be obtained. The first step in resolving the issue is this. The dimensional boundary layer is replaced by a non-dimensional version. The control equation is replaced with a similarity variable since the flow under investigation is not steady. The numerical solution used is the Keller Box method. Numerical data will be obtained to investigate the effects of radiation parameters and Stuart Number on the temperature and speed profiles.

II. RESEARCH METHODS

This section explains the research stages which include mathematical models, numerical simulations, and analysis of results.

2.1 Mathematical models

This step uses the governing equations to create a mathematical representation of the flow of viscous fluid. [12]. The steps of model development are given as follows.

1. Using governing equations derived from physics, one can create a three-dimensional equation of continuity, momentum, and energy.
2. Using nondimensional variables and nondimensional parameters, the resulting dimensional equations are transformed into nondimensional equations.
3. The radiation parameter and the Stuart number variables are used to substitute the resulting nondimensional equations.
4. Using similarity variables and flow functions, the derived nondimensional equations are simplified into a single variable for convenience of computation.

2.2 Numerical simulations

Proceed with numerical solutions at this point. With the aid of software, the Keller Box approach was applied in this study. One approach to solving parabolic equations, particularly boundary layer equations, is the Keller-Box method. The step sizes for time and space do not have to match in this implicit version, which has second order precision in both space and time. Parabolic partial differential equations can be solved more accurately and quickly with this.

The shape of a second-order or high-order differential equation is changed into a first-order equation when using the Keller-Box method [9].

2.3 Results analysis and discussions

At this stage, an analysis is carried out of the results of the influence of related parameters, namely the Stuart number and Radiation parameters on the viscosity fluid flow that passes through the cylinder surface.

III. MATHEMATICAL MODELS

Mathematical representations of viscous fluids that take radiation and the Stuart number into account. The First concept of Thermodynamics, Newton's Second Law, and the concept of conservation of mass are the sources of the building equations that were utilised to construct the model. The momentum equation is modified by adding the influence of the Stuart number parameter. In addition, modifications were also made to the energy and radiation equations. These two parameters will later be analysed regarding the effect of changes in speed and temperature on the viscosity of the fluid flow. Three building equations are formed from the outcomes of this derivation: the energy equation, the momentum equation, and the continuity equation.

Continuity equation:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0$$

Momentum equation – x axis:

$$\rho\left(\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right) = -\frac{\partial \bar{p}}{\partial \bar{x}} + \mu\left(\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right) - \sigma B_0^2 \bar{u} + \rho\beta(\bar{T} - T_\infty)g\bar{x}$$

Momentum equation – y axis:

$$\rho\left(\frac{\partial \bar{v}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{v}}{\partial \bar{y}}\right) = -\frac{\partial p}{\partial \bar{x}} + \mu\left(\frac{\partial^2 \bar{v}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{v}}{\partial \bar{y}^2}\right) - \sigma B_0^2 \bar{v} + \rho\beta(\bar{T} - T_\infty)g\bar{y}$$

Energy equation:

$$C_p\left(\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{T}}{\partial \bar{y}}\right) = c\left(\frac{\partial^2 \bar{T}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{T}}{\partial \bar{y}^2}\right) - \left(\frac{\partial \overline{q_r}}{\partial \bar{\mathrm{x}}} + \frac{\partial \overline{q_r}}{\partial \bar{\mathrm{y}}}\right)$$

with the boundary condition:

$$\bar{u} = \bar{v} = 0, \bar{T} = T_w \text{ when } \bar{y} = 0$$
$$\bar{u} = \bar{u}_e(\bar{x}), \bar{T} = T_\infty \text{ when } \bar{y} \to \infty$$

where $\bar{x}$ and $\bar{y}$ are the cartesian coordinate system in time ($\bar{t}$). Additionally, the dimensional variables of $\bar{u}$ and $\bar{v}$ represent the fluid velocity at $\bar{x}$ and $\bar{y}$ respectively.

To eliminate the dimensions of the parameters contained in the dimensionless mathematical model, the dimensional mathematical model was converted into a dimensionless mathematical model [13]. Considering the dimensionless variables transformation form, which is shown below

$$x=\frac{\bar{x}}{a}, y=Re^{1/2}\frac{\bar{y}}{a}, u=\frac{\bar{u}}{U_\infty}, v=Re^{1/2}\frac{\bar{v}}{U_\infty}, t=\frac{U_\infty\bar{t}}{a}, \overline{q_r}=\frac{-16\tau T_\infty^3\partial\bar{T}}{3k_1\partial\bar{y}}, p=\frac{\bar{p}}{\rho U_\infty^2}, T=\frac{\bar{T}-T_\infty}{T_w-T_\infty}$$

where

Re	: Reynolds Number
p	: pressure
T	: temperature
α	: convection parameter
Pr	: Prandtl Number

Nondimensional parameters defined as follow [12]: $St=\frac{\sigma^2B_0^2a}{\rho U_\infty}$ is Stuart Number parameter and $R=\frac{4\tau T_\infty^3}{k_1c}$ is defined Radiation Parameter.

Continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

Momentum equation – x axis:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{1}{Re}\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}-St\,u-\alpha T\,sinx$$

Momentum equation – y axis:

$$\frac{1}{Re}\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial x}+\frac{1}{Re^2}\frac{\partial^2v}{\partial x^2}+\frac{1}{Re}\frac{\partial^2v}{\partial y^2}-\frac{1}{Re}St\,v+\frac{1}{Re^{1/2}}\alpha Tcosx$$

Energy equation:

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{Pr}\frac{1}{Re^2}\frac{\partial^2T}{\partial x^2}+\frac{1}{Pr}\frac{\partial^2T}{\partial y^2}+\frac{1}{Pr}\frac{4R}{3}\frac{1}{Re^{1/2}}\frac{\partial^2T}{\partial x\partial y}+\frac{1}{Pr}\frac{4R}{3}\frac{\partial^2T}{\partial y^2}$$

Using the boundary layer approach where $Re\to\infty$ so $\frac{1}{Re}\to 0$, so only one momentum equation is used in building the model in the research. Therefore, this research uses a two-dimensional cross section or there are only two velocity components, namely u and v, whose flow is in the $x-y$ axis. To connect the two speed functions, a current function or flow function is introduced. This stream and similarity function is expressed as follows:

$$u=\frac{\partial\psi}{\partial y}, v=-\frac{\partial\psi}{\partial x}, \psi=t^{1/2}\mathrm{u}_e(x)f(x,\eta,t) \quad T=s(x,\eta,t) \quad \eta=\frac{y}{t^{1/2}}$$

Momentum Equation:

$$\frac{\partial^3f}{\partial\eta^3}+\frac{\eta}{2}\frac{\partial^2f}{\partial\eta^2}+t\frac{\partial\mathrm{u}_e}{\partial x}\left(1-\left(\frac{\partial f}{\partial\eta}\right)^2+f\frac{\partial^2f}{\partial\eta^2}\right)+St\,t\left(1-\frac{\partial f}{\partial\eta}\right)+\alpha ts\frac{sinx}{\mathrm{u}_e}=t\frac{\partial^2f}{\partial\eta\partial t}+t\mathrm{u}_e\left(\frac{\partial f}{\partial\eta}\frac{\partial^2f}{\partial\eta\partial x}-\frac{\partial f}{\partial x}\frac{\partial^2f}{\partial\eta^2}\right)$$

Energy Equation:

$$\frac{\partial^2s}{\partial\eta^2}+Pr\frac{\eta}{2}\frac{\partial s}{\partial\eta}+Prtf\frac{\partial\mathrm{u}_e}{\partial x}\frac{\partial s}{\partial\eta}+\frac{4R}{3}\frac{\partial^2s}{\partial\eta^2}=Prt\frac{\partial s}{\partial t}+Prt\mathrm{u}_e\left(\frac{\partial f}{\partial\eta}\frac{\partial s}{\partial x}-\frac{\partial f}{\partial x}\frac{\partial s}{\partial\eta}\right)$$

with boundary condition:

$$f = \frac{\partial f}{\partial \eta} = 0,\ s = 1 \text{ when } \eta = 0$$
$$\frac{\partial f}{\partial \eta} = 1,\ s = 0 \text{ when } \eta \to \infty.$$

IV. NUMERICAL SOLUTIONS

In this work, the equation model was numerically solved using the Keller-Box method. This method worked well and appropriately for solving boundary layer equations produced by parabolic partial differentials. The steps in this numerical solution were:

1. The system model equation was formed into a first order equation,
2. Discretization was carried out using the center difference method,
3. Linearization of the obtained equations using Newton's method and formed in a vector matrix, and
4. The result of linearization was solved by tridiagonal block matrix elimination technique.

V. RESULTS AND DISCUSSION

The results of the simulations that have been carried out show variation in Stuart Number (St) with velocity profiles (f') and temperature (s). In this simulation η is used, *step size* of $\eta, \Delta\eta = l_j = 0.1$ and time partition (t)*step size of time* $\Delta\eta = k^n = 0.05$.

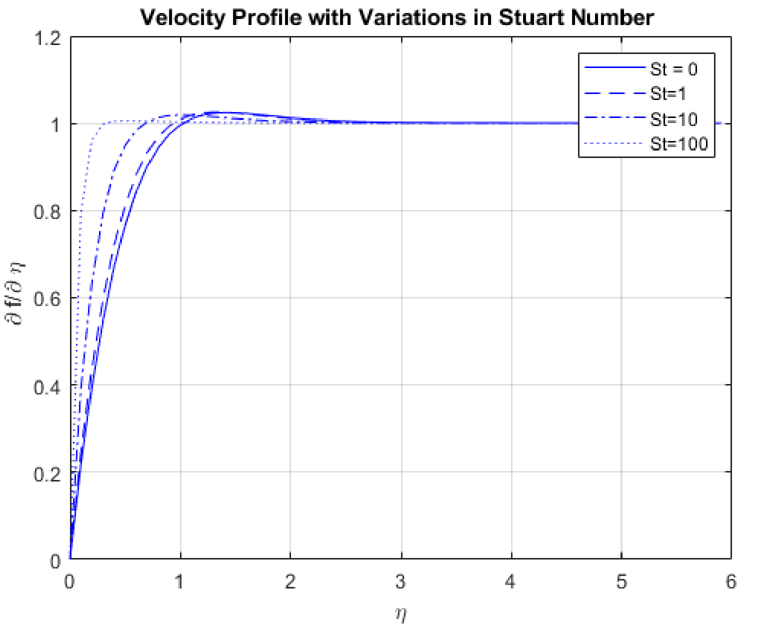

Figure 1. Variation of Stuart Numbers on Viscous Fluid Velocity

In Figure 1 shows that when $0 < \eta < 3$ the speed can be influenced by the Stuart number, whereas when η is greater than 3, whatever the value of the Stuart number does not affect the speed profile. The graphic results in Figure 1. show that the greater the Stuart number value, the greater the fluid flow speed. This is because the Lorentz force which acts becomes greater as the magnetic field increases which affects the viscous fluid, which can be shown systematically by $St = \frac{\sigma^2 B_0^2 a}{\rho U_\infty}$ which means that $St \sim B_0$. By increasing the Lorentz force, the movement of electric charges in the magnetic field increases and the momentum of this fluid increases.

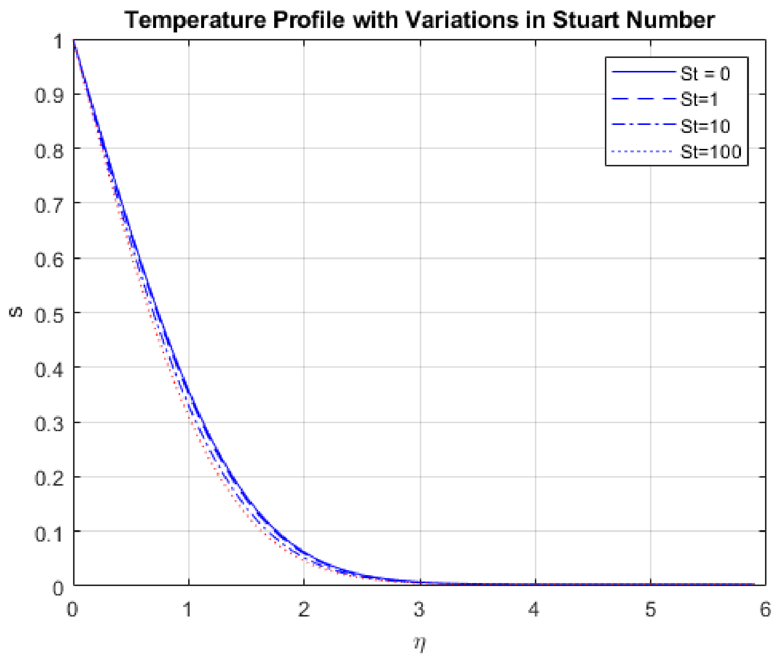

Figure 2. Variation of Stuart Numbers on Viscous Fluid Temperature

In Figure 2, the temperature is where the value decreases from s=1 to $s = 1$ when $0 < \eta < 4.5$, after more than 4.5 the temperature stabilizes towards $s = 0$ regardless of the magnetic parameter value. Apart from that, the Lorentz force caused by the presence of a magnetic field across the flow makes this fluid increase in internal energy. Internal energy is used for fluid particles to move according to the stream line, so that the fluid temperature will decrease with increasing magnetic field.

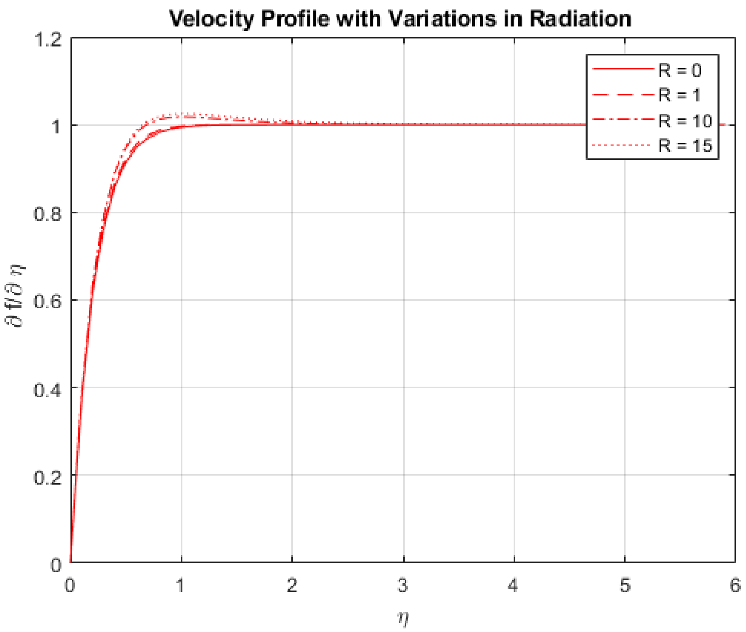

Figure 3. Variation of Radiation on Viscous Fluid Velocity

In Figure 3, demonstrates that variations in the viscous fluid flow velocity are independent of the radiation parameter. This figure shows that there is no discernible flow acceleration with increasing radiation parameter value.

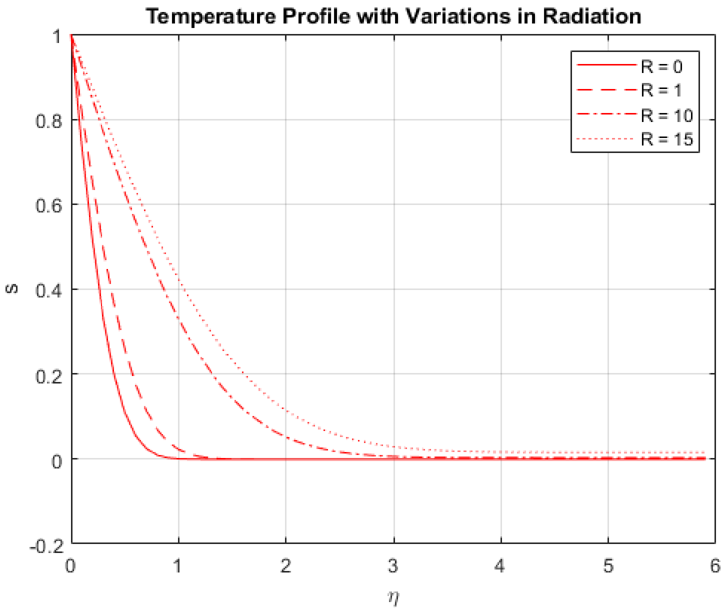

Figure 4. Variation of Radiation on Viscous Fluid Temperature

In Figure 4, the results show that the smaller the radiation value, the lower the resulting temperature profile. The temperature profile decreases when $0 < \eta < 3.7$ from $s = 1$ to $s = 0$. When η is more than 3.7, the resulting temperature profile, whatever the parameter variations, the value will remain stable. It is known that $R = \frac{4\tau T_\infty^3}{k_1 c}$ then $R \sim T_\infty^3$, where T_∞^3 is the environmental temperature, therefore the greater the environmental temperature, the faster the fluid flow will reach stability.

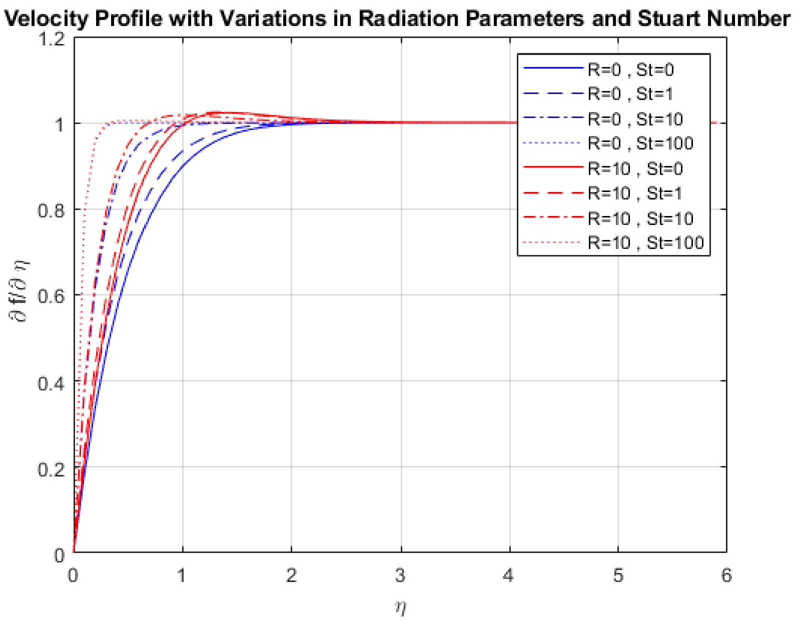

Figure 5. Variation of Stuart Numbers and Radiation on Viscous Fluid Velocity

In Figure 5, it is evident that increasing the Stuart number has an effect on the viscous fluid flow speed, indicating that the parameter value is not sufficiently significant to raise the viscous fluid flow speed.

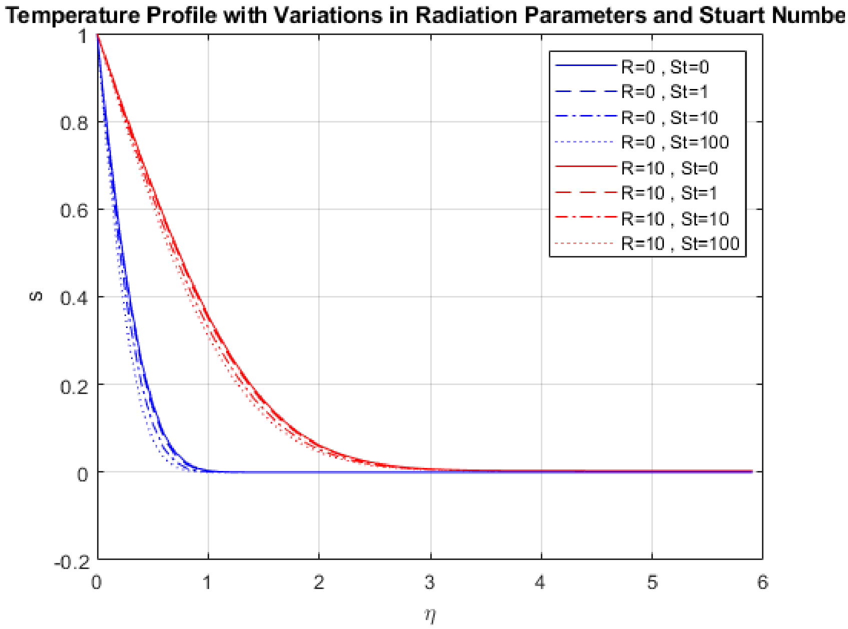

Figure 6. Variation of Stuart Numbers and Radiation on Viscous Fluid Temperature

In Figure 6, it can be seen that the fluid temperature decreases as the radiation parameter variation value increases. The temperature profile changes when $0 < \eta < 3$ starting from $s = 1$ to $s = 0$. When the η value is more than 3 the value does not change. Temperature is influenced by small magnetic and radiation values. It is known that $R = \frac{4\tau T_\infty^3}{k_1 c}$ then $R \sim T_\infty^3$, where T_∞^3 is the environmental temperature, therefore the greater the environmental temperature, the faster the fluid flow will reach stability.

VI. CONCLUSIONS

This study examines the effects of the radiation parameter and Stuart number on viscous fluid flowing through sphere surfaces. The viscous fluid flow is mathematically modelled using the momentum equation, energy equation, and continuity equation. The solution is obtained numerically using the Keller Box Method. The findings indicate that the viscous fluid velocity decreases significantly with decreasing radiation and Stuart number influence, but that the viscous fluid temperature increases due to viscosity.

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