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PYTHAGOREAN FUZZY SET AND ITS APPLICATION TO DETERMINING STUDENT CONCENTRATION USING MAX-MIN-MAX COMPOSITION

Imam Hafiidz Nuur[1], Arif Munandar[2*]

[1,2]Department of Mathematics, Sunan Kalijaga State Islamic University Yogyakarta,
Marsda Adisucipto, St. No 1, Yogyakarta, 55281, Indonesia
Email : [2]arif.munandar@uin-suka.ac.id
[*]Corresponding Author

Abstract. Pythagorean fuzzy set is a generalization of intuitionistic fuzzy set. As intuitionistic fuzzy set that can be used to help in solving problems regarding decision making, pythagorean fuzzy set can also be done for the same thing. In the pythagorean fuzzy set, a max-min-max composition relation will be formed and used it to solve decision-making problems. Through this research, decision making in determining the concentration for students of the Mathematics undergraduates program at Sunan Kalijaga State Islamic University Yogyakarta is discussed based on data on student grades in compulsory courses that have been taken by students until the 4th semester. Concentration that is in line with the interests and abilities is expected to facilitate the writing of the student's final project.
Keywords: Pythagorean fuzzy set, Max-min-max composition, Decision making

I. INTRODUCTION

The new concept of mathematical logic was first discovered by Lotfi A. Zadeh in the early 20th century [1] and was named fuzzy logic. Fuzzy logic is a logic system that allows levels of truth that are not only true or false, but also between true and false. Unlike Boolean logic whose computational approach is only based on "true or false", fuzzy logic uses a computational approach based on "degrees of truth". This allows fuzzy logic to form systems that have smoother sensors. Because of this ability, fuzzy logic can be used to solve control system problems as in [2], applied in image processing as in research [3], applications in decision making [4] and research in other fields as written in [5]. Fuzzy logic is not even only applied as a theory, but has been applied in the dick system on washing machines [6] or refrigerators as research [7]. The theory of fuzzy is also applied to other mathematical materials such as graphs. The discussion then becomes a separate topic called fuzzy graphs. Some research related to applications on fuzzy graphs are discussed in [8] and [9].

The development of fuzzy sets is characterized by the expansion of the properties of membership values. The formulation of the non-membership function is the complement of the membership function, the summation property of the membership and non-membership functions which is less than equal to 1. There is a value of doubt in the proposal of the idea so that the value of doubt is also taken into account. The background of the incident is a new structure, namely intuitionistic fuzzy set(IFS), introduced by Attanasov in [10] and developed in [11]. Since IFS is a development of fuzzy sets, it can be applied in similar fields. Research related to the application of IFS in the field of image processing can be found in [12], in the field of forecasting [13], and some other research in decision making such as [14] and [15]. Research on IFS from the point of view of algebraic structures has also received a lot of attention, for example, research in [16] and [17]. Due to the many applications of IFS, some researchers have summarized them in [18] and [19].

Intuitionistic fuzzy sets (IFS) must satisfy the property of connecting membership values (μ), non-membership values (v) and indecision values (π), such that $\mu + v \leq 1$ and $\mu + v + \pi = 1$. Because of this flexibility, Intuitionistic fuzzy sets (IFS) provide a more flexible framework to describe uncertainty and vagueness. The situation that there is a possibility that $\mu + v + \pi \geq 1$ encourages the development of IFS. Based on this, a new fuzzy set system called pythagorean fuzzy set (PFS) was developed. The constraints given in PFS are looser than IFS because the constraints of membership value (μ), non-membership (v) and indecision π fulfill the condition $\mu^2 + v^2 + \pi^2 = 1$. It is clear that there are possibilities where $\mu + v + \pi > 1$, but $\mu^2 + v^2 + \pi^2 = 1$, so it is quite clear that PFS is a generalization of IFS.

Pythagorean fuzzy sets introduced by Yager in [20] and [21] provide information about the expansion of the concept of intuitionistic fuzzy sets that allow membership values in a wider range than IFS, so as to describe uncertainty better. Because it is a generalization of IFS, PFS can also be applied as an application of fuzzy sets or IFS. Some further research on the application of PFS can be found in studies such as [22] and [23]. Another application on multi-criteria decision-making framework integrating Interval-Valued Pythagorean Fuzzy Sets (IVPFS) with TOPSIS (Technique for Order Preference by Similarity to an Ideal Solution) and GRA (Grey Relational Analysis) done by [24]. New distance measures for Pythagorean cubic fuzzy sets and its application done in research [25]. While research conducted by [26], discusses the concept of PFS and its application in career placement problems based on academic performance through a max-min-max composition approach used to evaluate and select the most suitable career option based on academic performance.

This research will use the concept of PFS with a max-min-max composition approach to help students in decision making. The decision making in this study is the determination of concentration for S-1 Mathematics Study Program Students of Sunan Kalijaga State Islamic University Yogyakarta based on the value of courses that have been taken up to semester 4. The use of max-min-max composition in pythagorean fuzzy sets is expected to increase the accuracy of decision making in determining concentration, making it easier for students to complete their final project.

II. METHODOLOGY

This research uses several definitions and theorems related to phytagorean fuzzy sets. Some of these theorems form the basis for the discussion of the methods used in this study. All results written as the basis of this theory are taken from research [26].

Definition 1. [26] *Given X is a universe set. Pythagorean fuzzy set A is the set of ordered pairs from X and is defined as follows*

$$A = \{(x, \mu_a(x), v_a(x)) | x \in X\}$$

With the function

$$\mu_a(x): X \to [0,1] \qquad v_a(x): X \to [0,1]$$

where $(\mu_a(x))$ *denotes the degree of membership and* $(v_a(x))$ *denotes the degree of nonmembership of element* $x \in X$ *of* A. *Then for each* $x \in X$ *holds*

$$0 \leq (\mu_a(x))^2 + (v_a(x))^2 \leq 1.$$

The degree of indeterminacy of $x \in X$ *over* A *is defined by*

$$\pi_a(x) = \sqrt{1 - [(\mu_a(x))^2 + (v_a(x))^2]}$$

with $\pi_a(x) \in [0,1]$. *So* $(\mu_a(x))^2 + (v_a(x))^2 + (\pi_a(x))^2 = 1$. *Pythagorean fuzzy set over* X *is denoted by* $PFS(X)$.

Theorem 1. [26] *Given a set* $X = \{x_i\}$ *as the universe, for* $i = 1,2, \ldots, n$ *and* $A \in PFS(X)$. *If* $\pi_a = 0$ *then this equation below is holds*

1. $\mu_a(x_i) = \sqrt{|(v_a(x_i) + 1)(v_a(x_i) - 1)|}$
2. $v_a(x_i) = \sqrt{|(\mu_a(x_i) + 1)(\mu_a(x_i) - 1)|}$.

Definition 2. [26] *Given any universal set* X. *If* $A \in PFS(X)$ *then the complement of* A *is denoted as* A^c *and defined as*

$$A^c = \{(x, v_a(x), \mu_a(x)) |\, x \in X\}.$$

Definition 3. [26] *Given Pythagorean fuzzy sets* $A, B \in PFS(X)$ *the following union and intersection operations defined as*

1. $A \cup B = \left\{\left(x, max\big(\mu_a(x), \mu_b(x)\big), min\big(v_a(x), v_b(x)\big)\right) |\, x \in X\right\}$
2. $A \cap B = \left\{\left(x, min\big(\mu_a(x), \mu_b(x)\big), max\big(v_a(x), v_b(x)\big)\right) |\, x \in X\right\}$.

Definition 4. [26] *Given Pythagorean fuzzy sets* $A, B \in PFS(X)$. *The multiplication and addition operations of* A *and* B *as PFS are expressed as follows*

1. $A \oplus B = \left\{\left(x, \sqrt{(\mu_a(x))^2 + (\mu_b(x))^2 - (\mu_a(x))^2 + (\mu_b(x))^2}, v_a(x) v_b(x) \;\right| x \in X\right\}$
2. $A \otimes B = \left\{\left(x, \mu_a(x)\mu_b(x), \sqrt{(v_a(x))^2 + (v_b(x))^2 - (v_a(x))^2 + (v_b(x))^2} \;\right| x \in X\right\}$.

As with the properties in ordinary sets related to intersection and combination operations, similar properties apply in fuzzy phytagorean sets. Especially with the presence of additional addition and multiplication operations in the PFS defined above.

Theorem 2. [26] *Given a set universe* X, *and* $A, B \in PFS(X)$. *The union, intersection and addition and multiplication operations defined above, have the following properties:*

1. *Idempotent*
   a. $A \cup A$
   b. $A \cap A$
   c. $A \oplus A$
   d. $A \otimes A$
2. *Commutative*
   a. $A \cup B = B \cup A$

   b. $A \cap B = B \cap A$
   c. $A \oplus B = B \oplus A$
   d. $A \otimes B = B \otimes A$
3. *Associative*
   a. $A \cup (B \cup C) = (A \cup B) \cup C$
   b. $A \cap (B \cap C) = (A \cap B) \cap C$
   c. $A \oplus (B \oplus C) = (A \oplus B) \oplus C$
   d. $A \otimes (B \otimes C) = (A \otimes B) \otimes C$
4. *Distributive*
   a. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
   b. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
   c. $A \oplus (B \cap C) = (A \oplus B) \cup (A \oplus C)$
   d. $A \otimes (B \cup C) = (A \otimes B) \cap (A \otimes C)$
   e. $A \otimes (B \cup C) = (A \otimes B) \cup (A \otimes C)$
   f. $A \oplus (B \cap C) = (A \oplus B) \cap (A \oplus C)$
5. *De Morgan Law*
   a. $(A \cap B)^c = A^c \cup B^c$
   b. $(A \cup B)^c = A^c \cap B^c$
   c. $(A \oplus B)^c = A^c \oplus B^c$
   d. $(A \otimes B)^c = A^c \otimes B^c$.

Inspired by the relation that arises in IFS as written in [19], the relation on pythagorean fuzzy set is defined as follows.

Definition 5. [26] *Given a function f for $A \in PFS(X)$ mapped to $B \in PFS(Y)$ such that it can be defined as follows:*

1. *Take any set A mapped by f, denoted by $f(A)$ which is a member of $PFS(Y)$*
$$\mu_{f(A)}(y) = \begin{cases} \vee_{x\in f^{-1}(y)}\, \mu_A(x), & for \quad f^{-1}(y) \neq \emptyset \\ 0, & for \quad f^{-1}(y) = \emptyset \end{cases}$$
   and
$$v_{f(A)}(y) = \begin{cases} \wedge_{z\in f^{-1}(y)}\, v_A(x), & for \quad f^{-1}(y) \neq \emptyset \\ 1, & for \quad f^{-1}(y) = \emptyset \end{cases}$$
2. *The inverse of the set B is mapped to the function f denoted by $f^{-1}\,(B)$ which is a member of $PFS(X)$:*
$$\mu_{f^{-1}(B)}(x) = \mu_B\big(f(x)\big)$$
   and
$$v_{f^{-1}(B)}(x) = v_B\big(f(x)\big).$$

Definition 6. [26] *Given an arbitrary set X, and $A \in PFS(X)$. The max-min-max composition of the fuzzy phytagorean set relation $R(X \Rightarrow Y)$ with $B \in PFS(Y)$ or can be written $B = R \circ A$, such that the degrees of membership and non-membership of B are*
$$\mu_B(y) = \vee\,(min[\mu_A\,(x), \mu_R\,(x,y)])$$
and for the degree of non-membership
$$v_B(y) = \wedge\,(max[v_A(x), v_R(x,y)]),$$
for each $(x, y) \in PFS(X, Y)$, with $\vee$=maximum and $\wedge$=minimum.

Definition 7. [26] *Given a pythagorean fuzzy set relation $Q(X \to Y)$ and $R(Y \to Z)$. The max-min-max composition of $R \circ Q$ is the pythagorean fuzzy set function relation from X to Z such that its membership values and non-membership value can be written as follows*

$$\mu_{R\circ Q}\,(x,z) = \vee\left(min\left[\mu_Q(x,y), \mu_R(y,z)\right]\right)$$
$$v_{R\circ Q}\,(x,z) = \wedge\left(max\left[v_Q(x,y), v_R(y,z)\right]\right).$$

Furthermore, the max-min-max composition value of $R \circ Q$ is calculated with the following formula

$$R \circ Q = \mu_{R\circ Q}\,(y) - v_{R\circ Q}(y)\pi_{R\circ Q}\,(y).$$

The following are research steps to determine the concentration of students that should be taken based on the grades obtained by these students in the courses that have been taken. The author summarizes these steps by paying attention to examples of application carried out in research [26]. In detail, the research steps that the author did are as follows:

1. This research begins by defining set P that is the set of student names and set S that it is the set of compulsory courses that students have obtained in semesters 1-4. The four fields of study are including Analysis(AN), Algebra(AL), Applied(APP), Statistics(STT), this set grouped in set D.
2. Constructing a fuzzy pythagorean relationship Q between the student's name and the value of compulsory courses, and written with $Q : P \to S$, then formed an R relationship between compulsory courses and concentration and written with $R : S \to D$.
3. Determine the membership value of the max-min-max composition relationship using Definition 8.
4. Determine the value of the composition relation $T = R \circ Q$ by using the formula $T = \mu_T - v_T\,\pi_T$, which is the formula written in Definition 9.
5. Determine the highest value of the relation T, if the highest value is not single, then the recommendation is selected based on the smallest π^2 with the formula
$$\pi_T{}^2 = \left(1 - (\mu_T{}^2 + v_T{}^2)\right).$$

III. RESULT AND DISCUSSION

This research began by giving questionnaires to students. There were 20 students who filled out questionnaires related to the grades obtained from compulsory courses until semester 4. The compulsory courses and their codes are as follows: Differential Calculus (DC), Mathematical Logic and Sets (MLS), Elementary Linear Algebra (ELA), Field Geometry (FG), Algorithms and Programming (AP), Integral Calculus (IC), Geometry of Space (GS), Linear Program (LP), Statistical Methods (SM), Multivariable Calculus (MC), Introduction to Algebraic Structures (IAS), Elementary Differential Equations (EDE), Discrete Mathematics (DM), Numerical Methods (NM), Probability Theory (PT), Advanced Calculus (AC), Introduction to Real Analysis (IRA), Partial Differential Equations (PDE), Linear Algebra (LA), Introduction to Mathematical Models (IMM), Introduction to Mathematical Statistics (IMS), Complex Variable Functions (CVF), Hisab Rukyat (HR), and Islamic Financial Mathematics (IFM). After obtaining the value data from each of these courses, the membership value (μ_Q) is then calculated by dividing the student's score (achievement index) in each subject by 4 (maximum value). The calculation of the non-membership value uses the formula $v_Q{}^2 = 1 - \mu_Q{}^2 - \mu_Q{}^2$.

Then from the data collection, Table 1. is formed. For each course, the first row shows the membership value (μ_Q) of the course for each student, while the next row shows the student's non-membership (inability) value (v_Q), with the uncertainty value $\pi_Q = 0.1$.

Table 1. Membership degree (μ) and non-membership (v) of Q relation.

Q	Student																			
Course	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
DC	0.75	0.94	1	0.81	0.81	0.01	0.98	0.14	0.94	0.98	0.01	0.72	0.29	0.57	0.82	0.71	0.64	0.57	0.57	0.75
	0.25	0.06	0	0.19	0.19	0.99	0.02	0.86	0.06	0.02	0.99	0.28	0.71	0.43	0.18	0.29	0.36	0.43	0.43	0.25
MLS	0.88	0.94	0.88	0.69	0.81	0.07	0.83	0.45	0.54	0.57	0.86	0.38	0.43	0.88	1	0.36	0.28	0.36	0.63	0.88
	0.13	0.06	0.13	0.31	0.19	0.93	0.17	0.55	0.46	0.43	0.14	0.62	0.57	0.12	0	0.64	0.72	0.64	0.37	0.13
ELA	0.63	0.75	0.94	0.69	0.69	0.78	0.18	0.62	0.34	0.22	0.36	0.33	0.17	0.62	0.42	0.30	0.26	0.72	0.03	0.63
	0.38	0.25	0.06	0.31	0.31	0.22	0.82	0.38	0.66	0.78	0.64	0.67	0.83	0.38	0.58	0.70	0.74	0.28	0.97	0.38
FG	0.88	0.88	0.81	0.88	0.88	0.5	0.92	0.34	0.06	0.54	0.52	0.04	0.99	0.31	0.38	0.39	0.90	0.22	0	0.88
	0.13	0.13	0.19	0.13	0.13	0.50	0.08	0.66	0.94	0.46	0.48	0.96	0.01	0.69	0.62	0.61	0.10	0.78	1	0.13
AP	1	1	1	1	0.94	0	0.19	0.21	0.03	0.39	0.61	0.85	0.09	0.79	0.86	0.8	0.96	0.96	0.51	1
	0	0	0	0	0.06	1	0.81	0.79	0.97	0.61	0.39	0.15	0.91	0.21	0.14	0.20	0.04	0.04	0.49	0
IC	0.81	0.81	0.81	0.88	0.88	0.88	0.83	0.70	0.81	0.91	0.97	0.98	0.65	1.00	0.65	0.27	0.44	0.75	0.70	0.88
	0.19	0.19	0.19	0.13	0.13	0.12	0.17	0.30	0.19	0.09	0.03	0.02	0.35	0.00	0.35	0.73	0.56	0.25	0.30	0.13
GS	0.81	0.81	0.81	0.81	0.88	0.77	0.10	0.70	0.10	0.87	0.44	0.39	0.24	0.17	0.16	0.37	0.59	0.27	0.91	0.81
	0.19	0.19	0.19	0.19	0.13	0.23	0.90	0.30	0.90	0.13	0.56	0.61	0.76	0.83	0.84	0.63	0.41	0.73	0.09	0.19
LP	0.75	0.69	0.75	0.75	0.81	0.92	0.96	0.27	0.47	0.42	0.72	0.30	0.18	0.67	0.90	0.30	0.29	0.71	0.44	0.75
	0.25	0.31	0.25	0.25	0.19	0.08	0.04	0.73	0.53	0.58	0.28	0.70	0.82	0.33	0.10	0.70	0.71	0.29	0.56	0.25
SM	0.94	0.94	0.94	0.94	0.81	0.62	0.64	0.18	0.83	0.68	0.94	0.47	0.29	0.03	0.92	0.63	0.96	0.77	0.42	0.94
	0.06	0.06	0.06	0.06	0.19	0.38	0.36	0.82	0.17	0.32	0.06	0.53	0.71	0.97	0.08	0.37	0.04	0.23	0.58	0.06
MC	0.44	0.69	0.75	0.50	0.56	0.02	0	0.47	0.71	0.77	0.73	0.17	0.99	0.78	0.56	0.32	0.43	0.90	0.44	0.50
	0.56	0.31	0.25	0.50	0.44	0.98	1	0.53	0.29	0.23	0.27	0.83	0.01	0.22	0.44	0.68	0.57	0.10	0.56	0.50
IAS	0.94	0.94	0.81	0.81	0.75	0.86	0.60	0.76	0.38	0.61	0.22	0.72	0.35	0.76	0.65	0.90	0.29	0.30	0.06	0.81
	0.06	0.06	0.19	0.19	0.25	0.14	0.40	0.24	0.62	0.39	0.78	0.28	0.65	0.24	0.35	0.10	0.71	0.70	0.94	0.19
EDE	1	1	1	1	1	0.20	0.40	0.39	0.60	0.66	0.84	0.93	0.31	0.59	0.27	0.51	0.10	0.28	0.70	1
	0	0	0	0	0	0.80	0.60	0.61	0.40	0.34	0.16	0.07	0.69	0.41	0.73	0.49	0.90	0.72	0.30	0
DM	0.88	0.94	1	0.94	0.94	0.95	0.41	0.45	0.62	0.48	0.29	0.18	0.50	0.79	0.09	0.73	0.36	0.65	0.80	0.94
	0.13	0.06	0	0.06	0.06	0.05	0.59	0.55	0.38	0.52	0.71	0.82	0.50	0.21	0.91	0.27	0.64	0.35	0.20	0.06
NM	0.75	0.75	0.81	0.75	0.81	0.20	0.13	0.88	0.33	0.33	0.53	0.87	0.67	0.03	0.51	0.73	0.57	0.77	0.05	0.75
	0.25	0.25	0.19	0.25	0.19	0.80	0.87	0.12	0.67	0.67	0.47	0.13	0.33	0.97	0.49	0.27	0.43	0.23	0.95	0.25
PT	0.81	0.81	0.81	0.81	0.81	0.22	0.97	0.60	0.66	0.04	0.24	0.16	0.63	0.95	0.30	0.87	0.72	0.12	0.17	0.81
	0.19	0.19	0.19	0.19	0.19	0.78	0.03	0.40	0.34	0.96	0.76	0.84	0.37	0.05	0.70	0.13	0.28	0.88	0.83	0.19
AC	1	1	1	1	1	0.64	0.55	0.72	0.02	0.98	0.47	0.45	0.17	0.20	0.88	0.39	0.24	0.83	0.68	1
	0	0	0	0	0	0.36	0.45	0.28	0.98	0.02	0.53	0.55	0.83	0.80	0.12	0.61	0.76	0.17	0.32	0
IRA	0	0	0	0	1	0.30	0.40	0.76	0.86	0.83	0.87	0.68	0.83	0.92	0.22	0.89	0.29	0.81	0.35	0
	1	1	1	1	0	0.70	0.60	0.24	0.14	0.17	0.13	0.32	0.17	0.08	0.78	0.11	0.71	0.19	0.65	1
PDE	1	1	1	0.94	1	0.74	0.08	0.69	0.98	0.69	0.75	0.70	0.31	0.19	0.86	0.58	0.49	0.30	0.33	0.94
	0	0	0	0.06	0	0.26	0.92	0.31	0.02	0.31	0.25	0.30	0.69	0.81	0.14	0.42	0.51	0.70	0.67	0.06
LA	0.69	0.75	0.75	0.75	0.75	0.97	0.50	0.69	0.26	0.15	1.00	0.77	0.13	0.49	0.03	0.44	0.35	0.08	0.69	0.75
	0.31	0.25	0.25	0.25	0.25	0.03	0.50	0.31	0.74	0.85	0.00	0.23	0.87	0.51	0.97	0.56	0.65	0.92	0.31	0.25
IMM	0.94	1	1	0.94	1	0.97	0.82	0.64	0.53	0.89	0.96	0.53	0.43	0.12	0.80	0.53	0.59	0.14	0.77	0.94
	0.06	0	0	0.06	0	0.03	0.18	0.36	0.47	0.11	0.04	0.47	0.57	0.88	0.20	0.47	0.41	0.86	0.23	0.06
IMS	0	0	0	0	1	0.02	0.35	0.22	0.90	0.91	0.71	0.29	0.92	0.20	0.81	0.97	0.46	0.82	0.54	0.00
	1	1	1	1	0	0.98	0.65	0.78	0.10	0.09	0.29	0.71	0.08	0.80	0.19	0.03	0.54	0.18	0.46	1.00
CVF	0.63	0.75	0.81	0.75	0.88	0.13	0.35	0.08	0.11	0.51	0.73	0.49	0.23	0.84	0.24	0.80	0.66	0.05	0.67	0.75
	0.38	0.25	0.19	0.25	0.13	0.87	0.65	0.92	0.89	0.49	0.27	0.51	0.77	0.16	0.76	0.20	0.34	0.95	0.33	0.25
HR	0	0	0	0	1	0.15	0.02	0.42	0.82	0.57	0.54	0.68	0.98	0.35	0.72	0.60	0.89	0.13	0.72	0.00
	1	1	1	1	0	0.85	0.98	0.58	0.18	0.43	0.46	0.32	0.02	0.65	0.28	0.40	0.11	0.87	0.28	1.00
IFM	0	0	0	0	1	0.18	0.69	0.86	0.51	0.50	0.64	0.77	0.58	0.95	0.87	0.06	0.01	0.38	0.23	0.00
	1	1	1	1	0	0.82	0.31	0.14	0.49	0.50	0.36	0.23	0.42	0.05	0.13	0.94	0.99	0.62	0.77	1.00

After the Q relation is formed, then a fuzzy pythagorean relation R will be formed. The fuzzy pythagorean relation R states the relationship of the course with the concentration in set D. This table shows information on the relationship and unrelatedness of a course to the concentration. Membership values and non-membership values in the table are determined hypothetically, based on the author's analysis and justification by considering the required concentration courses in each existing concentration. So that the membership matrix (μ) and non-membership (v) are obtained which can be shown in the form of the following table.

Table 2. Membership degree (μ) and non-membership (v) of R relation.

R		DC	MLS	ELA	FG	AP	IC	GS	LP	SM	MC	IAS	EDE	DM	NM	PT	AC	IRA	PDE	LA	IMM	IMS	CVF	HR	IFM
AN	μ	0.6	0.9	0.6	0.7	0.4	1	1	0.6	0.4	0.9	0.7	0.6	0.5	0.5	0.4	0.9	1	0.6	0.7	0.6	0.4	1	0.8	0.4
	v	0.63	0.18	0.63	0.5	0.83	0	0	0.63	0.83	0.18	0.5	0.63	0.74	0.74	0.83	0.18	0	0.63	0.5	0.63	0.83	0	0.35	0.83
AL	μ	0.5	0.9	1	0.6	0.7	0.6	0.6	0.7	0.5	0.7	1	0.6	0.7	0.6	0.5	0.6	0.8	0.6	1	0.7	0.5	0.7	0.6	0.5
	v	0.74	0.18	0	0.63	0.5	0.63	0.63	0.5	0.74	0.5	0	0.63	0.5	0.63	0.74	0.63	0.35	0.63	0	0.5	0.74	0.5	0.63	0.74
APP	μ	0.87	0.49	0.16	1	1	0.81	1	1	0.81	0.81	0.64	1	0.9	0.89	0.81	0.81	0.64	1	0.81	1	0.64	0.64	0.6	0.8
	v	0.23	0.75	0.96	0	0	0.33	0	0	0.33	0.33	0.58	0	0.18	0.2	0.33	0.33	0.58	0	0.33	0	0.58	0.58	0.63	0.35
STT	μ	1	0.7	0.2	0.2	0.76	0.6	0.23	0.6	1	0.5	0.32	0.6	0.5	0.5	1	0.5	0	0.5	0.32	0.6	1	0.5	0.1	0.8
	v	0	0.5	0.95	0.95	0.41	0.63	0.94	0.63	0	0.74	0.89	0.63	0.74	0.74	0	0.74	0.99	0.74	0.89	0.63	0	0.74	0	0.35

Each concentration has compulsory courses that must be taken so that students are required to pay attention to the ability and weight of grades in prerequisite courses taken from semester 1-4. For example, the Analysis concentration has compulsory courses in Spherical Trigonometry and Analytic Geometry with prerequisites in Spatial Geometry. The Algebra Concentration has compulsory courses in the concentration of Introduction to Number Theory, Set Theory, Fuzzy Logic which have prerequisite courses in Mathematical Logic and Sets.

After obtaining Table 2, then the fuzzy pythagorean composition relation $T = R \circ Q$ will be sown in Table 3. The first column is the result of the relation of the membership value ($\mu_{R\circ Q}$) and the second column is the result of the relation of the non-membership value $\left(v_{R\circ Q}\right)$.

Tabel 3. Membership degree (μ) and non-membership (v) of $R \circ Q$ relation.

$R \circ Q$		Student																			
Concentration		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
AN	μ	0.9	0.9	0.9	0.9	1	0.88	0.83	0.76	0.86	0.91	0.97	0.98	0.90	1	0.9	0.89	0.8	0.9	0.91	0.9
	v	0.18	0.18	0.18	0.13	0	0.12	0.17	0.24	0.14	0.09	0.03	0.02	0.17	0	0.18	0.11	0.34	0.18	0.09	0.13
AL	μ	0.94	0.94	0.94	0.81	0.81	0.97	0.83	0.76	0.8	0.8	1	0.77	0.8	0.88	0.9	0.9	0.7	0.8	0.7	0.88
	v	0.06	0.06	0.06	0.19	0.19	0.03	0.18	0.24	0.35	0.35	0	0.23	0.35	0.18	0.18	0.1	0.5	0.28	0.31	0.18
APP	μ	1	1	1	1	1	0.97	0.96	0.88	0.98	0.89	0.96	0.93	0.99	0.81	0.90	0.81	0.96	0.96	0.91	1
	v	0	0	0	0	0	0.03	0.04	0.2	0.02	0.11	0.04	0.07	0.01	0.21	0.10	0.20	0.04	0.04	0.09	0
STT	μ	0.94	0.94	1	0.94	1	0.62	0.98	0.80	0.94	0.98	0.94	0.77	0.92	0.95	0.92	0.97	0.96	0.82	0.63	0.94
	v	0.06	0.06	0	0.06	0	0.38	0.02	0.35	0.06	0.02	0.06	0.28	0.02	0.05	0.08	0.03	0.04	0.18	0.28	0.06

Table 4 is obtained through the following formula $T = \mu_T - v_T\pi_T$. Table 4 shows the suitability of what concentration students can take, meaning that the higher the T value in Table 4, the more suitable the concentration is.

Table 4. Relation T

T	Student																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
AN	0.829	0.829	0.829	0.848	1	0.826	0.736	0.611	0.789	0.881	0.957	0.981	0.831	0.998	0.829	0.842	0.632	0.822	0.875	0.848
AL	0.916	0.916	0.916	0.709	0.709	0.960	0.73	0.62	0.629	0.629	0.995	0.636	0.629	0.795	0.829	0.856	0.445	0.65	0.502	0.794
APP	1	1	1	1	1	0.962	0.954	0.789	0.973	0.844	0.948	0.903	0.986	0.695	0.85	0.702	0.947	0.942	0.875	1
STT	0.916	0.916	1	0.916	1	0.357	0.971	0.629	0.922	0.976	0.922	0.608	0.909	0.929	0.886	0.969	0.952	0.724	0.423	0.916

After Table 4 is formed, a degree of doubt table π_T will be formed to consider concentration recommendations if the highest T value is not unique. Based on Table 4 above, students 1,2,3,4,5 have the highest T value in Applied concentration so it is recommended to take Applied concentration. Meanwhile, student 19 has the same value in Applied and Analysis concentrations, therefore to determine the suitable concentration, the value of doubt is considered in Table 5. Because the value of doubt is higher in analysis, it is recommended that the student take the applied concentration.

Tabel 5. Degree of doubt table (π_T)

π_T	Student																			
Concentration	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
AN	0.4	0.4	0.4	0.42	0	0.46	0.53	0.61	0.49	0.4	0.26	0.18	0.4	0.07	0.4	0.44	0.5	0.41	0.45	0.42
AL	0.34	0.34	0.34	0.55	0.55	0.25	0.53	0.6	0.49	0.49	0.1	0.59	0.49	0.45	0.40	0.43	0.51	0.53	0.64	0.45
TER	0	0	0	0	0	0.24	0.26	0.44	0.21	0.44	0.28	0.36	0.15	0.55	0.43	0.55	0.28	0.29	0.40	0
STAT	0.34	0.34	0	0.34	0	0.69	0.22	0.49	0.33	0.2	0.33	0.58	0.4	0.32	0.39	0.22	0.27	0.54	0.72	0.34

If then compared with the reality that occurs, there are 14 students who take the concentration in accordance with the results suggested in the max-min-max composition relation. While the rest of the students took the concentration not in accordance with the results of the relationship. Some factors that influence students in determining the concentration are the closeness of students to the lecturer of the concentration, personal factors such as the patience of the lecturer, the ability factor in understanding the materials delivered by the lecturer.

IV. CONCLUSION

Through the phytagorean fuzzy set and by using max-min-max composition, a system can be formed that can help students in making decisions, namely determining their scientific concentration. The system that was formed was tested to be quite effective in determining concentration. This is evidenced by the presence of 70% of students whose concentration is in accordance with the results of recommendations from the system. The discrepancy of the system results can be understood as the impact of student interests that may consider personal closeness to certain concentration lecturers.

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