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ALGEBRAIC STRUCTURES IN HEREDITY HUMAN BLOOD GROUP SYSTEM

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Abstract. Marriage or in this case the researcher calls it "cross-operation" between two individuals (male and female) who have the same or different blood type has the probability to produce children (offspring) with the same blood type as one of the parents or even have a completely different blood type with both of them, whether it is the ABO blood type system or MN if it is associated with the rhesus system or not. The cross-operation between two individuals can be viewed from a mathematical perspective as an algebraic structure with one closed binary operation (OB). The cross-operation of ABO blood group system is an algebraic structure in groupoid form. The cross-operation of MN blood group system is an algebraic structure in groupoid form. And finally, the cross-operation of ABO and MN blood group systems when associated with the rhesus blood group system is an algebraic structure in groupoid form.
Keywords: algebraic structure, cross operation, heredity, blood group system.

1. INTRODUCTION

In science, humans cannot be separated from mathematics. This is because mathematics is the foundation for other sciences [1]. Mathematics also has a fairly broad role or application to science such as in the fields of biology, chemistry, physics, astronomy, economics, geography, information engineering and so on. Especially in biology, mathematics also has many applications and one of them is the application of algebraic structures in the field of human blood type systems.

An algebraic structure is a set that has certain (non-empty) elements, for example the set is S and is accompanied by one or more binary operations (OB) [2]. Where binary operations (OB) on a non-empty set are all functions that have a domain $S \times S$ and a codomain S [3]. Algebraic structure is divided into two, namely algebraic structure I which generally studies group theory and algebraic structure II which studies ring theory [2]. In algebraic structures, we learn about groups, semigroups, Abelian groups, cyclic groups, monoids, rings, fields, and so on. In this study, it will only be discussed about algebraic structure I, especially groupoids, semigroups, monoids, and groups and their application in human blood type heredity. So based on the above explanation, it can be concluded that algebraic structure I is a non-empty set S ($S \neq \emptyset$) equipped by one binary operation (OB).

Meanwhile, on the other hand, life in this world, all living things including humans need a life partner so that their lineage is well maintained. Each couple has differences and or similarities that can affect their future offspring. One of the factors that can affect offspring is genetics. Genetics has its roots in the Latin language, genes, which means a clan or clan. Genetics is one of the many branches of biology that studies the decline of human nature (heredity) [4]. Blood type is one of the many genetic traits that can be inherited /

passed down by humans (generations) to their offspring [5]. As we all know that the human blood group system is classified into several types consisting of the main group system and the non-major blood group system.

The human blood group system was first introduced by a scientist named Karl Landsteiner in 1901 [6]. He classified blood types and named the ABO blood type system [7]. In the ABO system, human blood is classified into 4 types, namely blood types A, B, AB and O [8]. The difference between the four blood types is in accordance with the antigens [9]. These antigens can be proteins, carbohydrates, glycoproteins and also glycolipids. An antigen can trigger the emergence of a special characteristic in the blood so that an antibody will respond. So it can be said that the presence or absence of antigens and antibodies in the blood determines the difference in the blood group system [5]. Where antigen is a compound that provides a stimulus to antibodies in the human body. Furthermore, antibodies will react to these stimuli so that the blood will clot.

There are as many as three alleles (versions) of the ABO blood group system. They are allele A (I^A), allele B (I^B) and allele O (I^O) [10,11], each of which has the genotype I^AI^A or I^AI^O, I^BI^B or I^BI^O and I^OI^O respectively [12]. O is recessive to both A and B types. Humans as diploid organisms, carry a double set, so human blood type is determined by two pairs of alleles with a probability of six combinations namely AA, BB, AB, OA, OB, and OO [11]. Since A and B are dominant to O, the genotypes AA and AO and BB and BO sequentially describe blood type A or phenotype A and blood type B or phenotype B [11]. I^A is not dominant to I^B and neither is I^B dominant to I^A. Since I^A and I^B do not dominate each other, I^A and I^B are codomain.

Then in 1927 Karl Landsteiner developed the blood group system into the MN blood group and was developed again by him in 1941 into the Rhesus blood group system consisting of positive Rhesus (Rh^+) and negative Rhesus (Rh^-) [7]. Rhesus positive (Rh^+) is a blood group that has Rh-Antigen in its erythrocytes. Meanwhile, negative Rhesus (Rh^-) is the opposite [13]. The Rh blood group system has two alleles, namely Rh and rh [10]. In the Rh blood group system, human blood groups are classified into eight types, namely blood group A (Rh^+ and Rh–), blood B (Rh^+ and Rh^-), blood AB (Rh^+ and Rh^-), and O (Rh^+ and Rh^-) [13].

While the MN blood group system is classified into M, N and MN [7]. Blood group M has M antigens, blood group N has N antigens, while for blood group MN has M and N antigens. Blood group M has the L^M allele (L^ML^M genotype), blood group N has the L^N allele (L^NL^N genotype) and blood group MN has the L^M allele and the L^N allele (L^ML^N genotype).

It is important to know that the blood group system in humans is actually classified into several types, namely the ABO system, MN system, P system, Colton (Co), Scianna (Sc), Rhesus (Rh), Lutheran (Lu), Lewis (Le), Kell (K), S / S factor, Duffy (Fy), Kidd (Jk), Diago (Di), Yt system, Xg, Domrock (Do), and so on [14]. This research will discuss the study of algebraic structures in the heredity of the ABO, MN, and Rhesus blood type systems.

Some previous studies that examine algebraic structures in blood group heredity are research conducted by [7] entitled "Implementation of Algebraic Structures in Blood Group Crosses", the research has some similarities with this research because it has the same topic of study. But the difference is, in this study it is studied in more detail and in depth about groupoids and semigups in the results and discussion section. In addition, in this study there is also a review of non-major blood groups. The research entitled "Algebraic Structure in Blood Type Inheritance" by [5], also has the same topic of study as this research, the difference lies in the discussion of algebra and also a review of non-major blood groups.

The gap found in this research and in previous research (relevant) is that it has not been able to determine the algebraic structure that exists in non-major blood groups. This is due to the limitations of literature sources such as journals, books, and so on. The purpose of this research is to find out the study of algebraic structures in biology, especially in the field of blood group heredity in the ABO, MN, and rhesus systems, both positive and negative.

II. METHODOLOGY

This research is a type of pure mathematics research with data collection methods, namely literature studies. Where researchers obtain the necessary data from several relevant studies such as journals, books, and so on. Studies relevant to this research such as studies in the field of algebraic structures, genetics, and the human blood type system.

1. Groups In Algebraic Structures And Blood Groups

a. Algebra

Algebra is a technique to symbolise a general description of things such as numbers, relations, functions, inverse functions, limit values and so on [5]. Problems in algebra are symbolised with certain symbols so that the problem becomes simple. The symbols used can be letters or special symbols whose use has been agreed upon and accepted by scientific circles. These symbols are useful as a substitute for numbers or constants. The branch of algebra that discusses and studies algebraic structures is called abstract algebra [15].

b. Algebraic Structure

Algebraic structure is a branch of mathematics that studies the structure of numbers. The study or discussion in algebraic structure is about a non-empty set accompanied by one or more binary operations (OB) [16]. The things discussed in the algebraic structure include the theory of groupoids, semigroups, monoids, subgroups, rings, fields, ideals, homomorphisms, isomorphisms, and so on.

A groupoid is an algebraic structure with one OB that is closed in nature. Semigroup is an algebraic structure with one OB that is closed and associative. Monoida is an algebraic structure with one OB that is closed, associative, and has an identity element, and group is an algebraic structure with one OB that is closed, associative, has an identity element and has an inverse [17]. In this study will be discussed specifically about the group and its application in the inheritance of blood groups in humans.

c. Groups and their Generalisations

Definition: According to [17] a group $(G, *)$ is a non-empty set G and is associated with an operation $*$ on G, such that the following axioms are satisfied:

1) For all $a, b \in G$, $a * b \in G$ (closedness property holds)
2) For all $a, b, c \in G$, $(a * b) * c = a * (b * c)$ (associative property holds).
3) There exists $i \in G$ such that for every $a \in G$, $i * a = a * i = a$ (has an identity element).
4) For every $a \in G$, there exists $a^{-1} \in G$ such that $a * a^{-1} = a^{-1} * a = i$ (has an inverse element).

Or symbolically written as follows:

1) $\forall\, a, b \in G,\ a * b \in G$
2) $\forall\, a, b, c \in G,\ (a * b) * c = a * (b * c)$
3) $\exists\, i \in G \ni \forall\, a \in G,\ i * a = a * i = a$
4) $\forall\, a \in G,\ \exists\, a^{-1} \in G \ni a * a^{-1} = a^{-1} * a = i$

So based on the explanation in point 3.2 above, the groupoid only applies property 1). In semigroups only properties 1) and 2) apply. In monoida only applies properties 1), 2) and 3). While the group applies properties 1) to properties 4).

Example:

a. The set of real numbers $\mathbb{R}$ with addition operation $(\mathbb{R}, +)$ is a group.
b. The set of integers Z to the addition operation (Z, +) is a group.
c. The set S8 = {1, 3, 5, 7} with multiplication operation (S8, ×) is a group.
d. The set of matrices of order 2×2 and det.(A) ≠ 0 with matrix multiplication operation is a group.

d. Blood Type System

i. Main Blood Type System

The main blood group system in humans is shown in the Table 1.

Table 1. *Main Blood Type System*

ISBT NUMBER	NAME	SYMBOL	ANTIGEN NUMBER	MAIN ANTIGEN	GENE NAMES	CHROMOSOME LOCATION NUMBER
001	ABO	ABO	4	A,B,A_1B,A_1	ABO	9
002	MNS	MNS	43	M,N,S,s,U	GYPA,GYPB,GYPE	4
003	P	P1	1	P_1	P1	22
004	Rhesus	Rh	49	D,C,E,c,e	RhD,RhCE	1
005	Lutheran	Lu	20	Lu^a,Lu^b	LU	19
006	Kell	K	25	K,k,Kp^a,Kp^b,Js^a,Js^b	KEL	7
007	Lewis	Le	6	Le^a,Le^b	FUT3	19
008	Duffy	Fy	6	Fy^a, Fy^b,Fy3	FY	1
009	Kidd	Jk	3	Jk^a,Jk^b,Jk3	SLC14A1	18

Source: [14]

ii. Other Blood Type Systems

The other blood group systems are shown in Table 2.

Table 2. *Other Blood Type Systems*

ISBT NUMBER	NAME	MAIN ANTIGEN	CHROMOSOME LOCATION NUMBER
010	Diego	Di^a,Di^b,Wr^a,Wr^b	17
011	Sistem Yt	Yt^a,Yt^b	7
012	Xg	Xg^b	X
013	Scianna	Sc1,Sc2	1
014	Dombrock	Do^a, Do^b,Hy,Gy^a,Jo^a	12
015	Colton	Co^a,Co^b,Co3	7
016	Landsteiner-Weiner	LW	19
017	Chido/Rodgers	CH/RG	6
018	H	H	19
019	Kx	Kx	X
020	Gerbich	Ge2,Ge3,Ge4	2
021	Cromer	Cr^a	1
022	Knops	Kn^a,Kn^b	1
023	Indian	In^a,In^b	11
024	Ok	Ok^a	19
025	Ralph	MER2	11
026	John Milton Hagen	JMH	15
027	I	I	6
028	Globoside	P	3
029	Gill	GIL	9

Source: [14]

iii. Blood Type ABO, MN and Rhesus Systems

In this study, researchers will focus more on the ABO, MN and Rhesus blood group systems. Below, Table 3 shows blood groups with the ABO, MN and Rhesus systems.

1) ABO Blood Type System

Karl Landsteiner discovered the ABO blood group system in 1901. This blood group system is divided into blood groups A, B, AB and O [7]. This blood group system includes A antigens and B antigens as well as A antibodies and B antibodies contained in the blood [18]. In general, the alleles and genotypes of this blood group system can be written as shown in Table 3.

Table 3. *ABO System Blood Group Alleles and Genotypes*

PHENOTYPE	ALLELES	GENOTYPE	DESCRIPTION
A	I^A	I^AI^A, I^AI^O	Homozygous A and Heterozygous A blood types
B	I^B	I^BI^B, I^BI^O	Homozygous B and Heterozygous B blood types
AB	I^A and I^B	I^AI^B	AB Blood Type
O	I^O	I^OI^O	O Blood Type

Source: [12, 19]

2) MN Blood Type System

The MN blood group system was also discovered by Karl Landsteiner in 1927, where this blood group system is divided into M, N and MN blood groups [7]. The MN blood group system is determined by two alleles, L^M and L^N, so there are three probabilities of the resulting phenotype. From the cross between two heterozygotes, a phenotypic ratio of 1:2:1 is obtained [20]. In general, the alleles and genotypes of the MN blood group system are written as shown in Table 4.

3) Rhesus Blood Group System

The Rhesus blood group system was also discovered by the same scientist who invented the ABO and MN Systems, namely Karl Landsteiner in 1941. The Rhesus blood group system consists of positive Rhesus (Rh^+) and negative Rhesus (Rh^-) [7]. The gene that is very decisive in the Rhesus blood group system (Rh) is influenced by one gene composed of two alleles R and r. Where positive rhesus (Rh^+) has the RR or Rr gene, while negative rhesus (Rh^-) has the rr gene which does not have Rh antigen. Marriage between two individuals of the positive Rhesus type (Rh^+) will definitely produce offspring of the positive Rhesus type (Rh^+) as well as applies to two individuals of the negative Rhesus type (Rh^-) it will produce offspring of the negative Rhesus type (Rh^-). Whereas if two individuals are of different Rhesus types (Rh^+ and Rh^- or Rh^- and Rh^+) then the probability of the offspring being of the Rh^+ type is 50% and of the Rh^- type is also 50%. The genotype and gamete writing of the Rhesus blood group system is shown in Table 5.

Table 4. *Blood Group Alleles and Genotypes of MN System*

PHENOTYPE	ALLELES	GENOTYPE	DESCRIPTION
M	L^M	$L^M L^M$	M Blood Type
N	L^N	$L^N L^N$	N Blood Type
MN	L^M and L^N	$L^M L^N$	MN Blood Type

Source: [21]

Table 5. *Rhesus Blood Type*

PHENOTYPE	GENOTYPE	GAMETE TYPE	DESCRIPTION
Rh^+	$I^{Rh} I^{Rh}$, $I^{Rh} I^{rh}$ or $Rh^+ Rh^+$, $Rh^+ rh^-$	I^{Rh} , I^{Rh}	Positive Rhesus
Rh^-	$I^{rh} I^{rh}$, $I^{Rh} I^{rh}$ or $rh^- rh^-$, $Rh^+ rh^-$	I^{rh}	Negative Rhesus

Source: [7]

Furthermore, the cross-operation between Rhesus positive and Rhesus negative blood types is shown in Table 6.

Table 6. *Rh^+ and Rh^- Blood Cross Operation*

$\otimes$	**Rh^+**	**Rh^-**
Rh^+	Rh^+	Rh^+ or Rh^-
Rh^-	Rh^+or Rh^-	Rh^-

Source: [7]

Both positive and negative Rhesus blood types can be associated with the ABO group system, as it is known that the ABO system blood type consists of blood A, B, AB and O will become A Rh^+, A Rh^-, B Rh^+, B Rh^-, AB Rh^+, AB Rh^-, O Rh^+ and O Rh^- [11, 13]. Meanwhile, if it is associated with the MN system blood type which is divided into M, N and MN, it will become M^+, M^-, N^+, N^-, MN^+ and MN^-.

III. RESULTS AND DISCUSSION

1. ABO Blood Type System

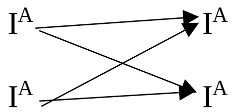
Suppose there are two people who want to get married, say Ani and Budi. Suppose the couple has the probability of the ABO blood group system (A, B, AB, and O). If the ABO blood type system is considered as a set, for example the set Y, then the members of Y are p = A, q = B, r = AB and s = O or can be written Y = {p, q, r, s}, and suppose (Y, ⊗) states the cross operation that occurs between members of Y. If the two have become a married couple and want to have children then the possibility of their child's blood type is shown in Table 7.

Table 7. *ABO Blood Type System Cross* Operation

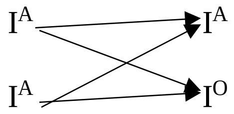
(Y, ⊗)	**A (I^AI^A, I^AI^O)**	**B (I^BI^B, I^BI^O)**	**AB (I^AI^B)**	**O (I^OI^O)**
A (I^AI^A, I^AI^O)	I^AI^A, I^AI^O, I^OI^O	I^AI^B,I^AI^O, I^BI^O,I^OI^O	I^AI^A,I^AI^B, I^AI^O,I^BI^O	I^AI^O, I^OI^O
B (I^BI^B, I^BI^O)	I^AI^B,I^AI^O, I^BI^O,I^OI^O	I^BI^B,I^BI^O,I^OI^O	I^AI^B,I^BI^B, I^AI^O,I^BI^O	I^BI^O, I^OI^O
AB (I^AI^B)	I^AI^A,I^AI^O, I^BI^O,I^AI^B	I^AI^B,I^BI^B, I^AI^O,I^BI^O	I^AI^A, I^AI^B, I^BI^B	I^AI^O, I^BI^O
O (I^OI^O)	I^AI^O, I^OI^O	I^BI^O, I^OI^O	I^AI^O , I^BI^O	I^OI^O

Source: [11, 19, 21]

The explanation for table 7 above is as follows:
Probability 1: Suppose Ani and Budi both have homozygous blood type A (I^AI^A), then the probability of their child is as follows:
$I^AI^A \otimes I^AI^A$ or can be written:

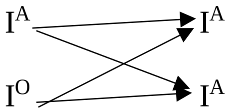


Therefore, one hundred per cent of the children born will be blood type A heterozygotes or genotype I^AI^A (I^AI^A, I^AI^A, I^AI^A, I^AI^A).
Probability 2: Suppose Ani and Budi are homozygous A and heterozygous A respectively, then the probability of a child being born is as follows:
$I^AI^A \otimes I^AI^O$ or can be written:

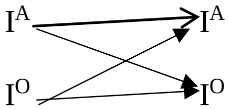


The child born from Ani's womb is either genotyped I^AI^A, I^AI^O, I^AI^A, or I^AI^O (I^AI^A or I^AI^O). In other words, the probability of I^AI^A and I^AI^O is 50% (equal).

Probability 3: Suppose Ani and Budi are blood type A heterozygous and A homozygous respectively, then the probability of a child being born is as follows:
$I^AI^O \otimes I^AI^A$ or can be written:



Then the child born is likely to have the genotype I^AI^A, I^AI^A, I^AI^O or I^AI^O (I^AI^A or I^AI^O) or the probability level is the same at 50%.
Probability 4: Suppose Ani and Budi each have the same blood type with the same genotype of A heterozygous, then the probability of a child being born is as follows:
$I^AI^O \otimes I^AI^O$ or can be written:



Then the child born may have the genotype I^AI^A, I^AI^O, I^AI^O, or I^OI^O or in other words, the probability of a child born with the I^AI^O genotype is 50%, I^AI^A and I^OI^O have the same probability level of 25%. So the possibility of a child being born is genotyped I^AI^A, I^AI^O or I^OI^O if the couple has either homozygous or heterozygous blood type A. Furthermore, the rule also applies to other ABO system blood group types.

If we look at table 7 carefully, then every element of Y is closed to the $\otimes$ operation, because every element resulting from the cross operation above also produces an element of Y. However, the cross operation above is not associative. To prove this, we can take any element of Y, for example p, q, r $\in$ Y. Then suppose p = A, q = B and r = O, then the results of the $\otimes$ operation that we can get from the three with different rules (order) are as follows: $p \otimes (q \otimes r) = A \otimes (B \otimes O) = (I^AI^A, I^AI^O) \otimes ((I^BI^B, I^BI^O) \otimes (I^OI^O)) = (I^AI^A, I^AI^O) \otimes (I^BI^O, I^BI^O, I^BI^O, I^BI^O, I^BI^O, I^BI^O, I^OI^O, I^OI^O) = (I^AI^A, I^AI^O) \otimes (I^BI^O, I^OI^O) = (I^AI^B, I^AI^O, I^AI^B, I^AI^O, I^AI^O, I^AI^O, I^AI^O, I^AI^O, I^AI^B, I^AI^O, I^BI^O, I^OI^O, I^AI^O, I^AI^O, I^OI^O, I^OI^O) = (I^AI^O, I^BI^O, I^AI^B, I^OI^O) =$ (I^AI^O or I^BI^O or I^AI^B or I^OI^O) = A is heterozygous or B is heterozygous or AB or O. Next we will find the result of the cross operation of $(p \otimes q) \otimes r = (A \otimes B) \otimes O = ((I^AI^A, I^AI^O) \otimes (I^BI^B, I^BI^O)) \otimes (I^OI^O) = (I^AI^B, I^AI^B, I^AI^B, I^AI^B, I^AI^B, I^AI^O, I^AI^B, I^AI^O, I^AI^B, I^AI^B, I^BI^O, I^BI^O, I^AI^B, I^AI^O, I^BI^O, I^OI^O) \otimes (I^OI^O) = (I^AI^B, I^AI^O, I^BI^O, I^OI^O) \otimes (I^OI^O) = (I^AI^O, I^AI^O, I^BI^O, I^BI^O, I^AI^O, I^AI^O, I^OI^O, I^OI^O, I^BI^O, I^BI^O, I^OI^O, I^OI^O, I^OI^O, I^OI^O, I^OI^O, I^OI^O) = (I^AI^O, I^BI^O, I^OI^O) =$ (I^AI^O or I^BI^O or I^OI^O) = A is heterozygous or B is heterozygous or O.

So based on the description above, it can be concluded that $p \otimes (q \otimes r) = A \otimes (B \otimes O) \neq (p \otimes q) \otimes r = (A \otimes B) \otimes O$. Therefore, the algebraic structure applicable in (Y, $\otimes$) is GRUPOID.

2. MN Blood Type System

The MN blood group system consists of M, N and MN as explained in the previous explanation. Suppose the MN blood type system is considered as a set W, whose members are a = M, b = N and c = MN or can be written as W = {a, b, c}. If the cross operation $\otimes$ applies to the set W, then W is crossed with itself and denoted by (W, $\otimes$), then the possible cross operation results are shown in Table 8.

Table 8. *Cross Operation of MN Blood Type System*

(W, ⊗)	**M (L^ML^M)**	**N (L^NL^N)**	**MN (L^ML^N)**
M (L^ML^M)	L^ML^M, L^ML^M, L^ML^M, L^ML^M	L^ML^N, L^ML^N, L^ML^N, L^ML^N	L^ML^M, L^ML^N, L^ML^M, L^ML^N
N (L^NL^N)	L^ML^N, L^ML^N, L^ML^N, L^ML^N	L^NL^N, L^NL^N, L^NL^N, L^NL^N	L^ML^N, L^ML^N, L^NL^N, L^NL^N
MN (L^ML^N)	L^ML^M, L^ML^M, L^ML^N, L^ML^N	L^ML^N, L^ML^N, L^NL^N, L^NL^N	L^ML^M, L^ML^N, L^ML^N, L^NL^N

Source: [7, 19]

In table 8 above, the set W is closed to the cross operation ⊗. To show whether the operation is associative or not, we can take any a = M, b = N and c = MN ∈ W such that: a ⊗ (b ⊗ c) = M ⊗ (N ⊗ MN) = (L^ML^M) ⊗ (L^ML^N, L^ML^N, L^NL^N, L^NL^N) = {(L^ML^M ⊗ L^ML^N, L^ML^M ⊗ L^ML^N, L^ML^M ⊗ L^NL^N, L^ML^M ⊗ L^NL^N)} = {L^ML^M, L^ML^N, L^ML^M, L^ML^N, L^ML^M, L^ML^N, L^ML^M, L^ML^N, L^ML^N, L^ML^N, L^ML^N, L^ML^N, L^ML^N, L^ML^N, L^ML^N, L^ML^N} = {L^ML^M, L^NL^N, L^ML^N}= {M, MN}. The next step we will look for (a ⊗ b) ⊗ c = (M ⊗ N) ⊗ MN = (L^ML^M ⊗ L^NL^N) ⊗ (L^ML^N) = {L^ML^N, L^ML^N, L^ML^N, L^ML^N} ⊗ {L^ML^N} = {L^ML^M, L^ML^N, L^ML^N, L^NL^N, L^ML^M, L^ML^N, L^ML^N, L^NL^N, L^ML^M, L^ML^N, L^ML^N, L^NL^N, L^ML^M, L^ML^N, L^ML^N, L^NL^N} = {L^ML^M, L^ML^M, L^ML^M, L^ML^M, L^NL^N, L^NL^N, L^NL^N, L^NL^N, L^ML^N, L^ML^N, L^ML^N, L^ML^N, L^ML^N, L^ML^N, L^ML^N, L^ML^N} = {L^ML^M, L^NL^N, L^ML^N} = {M, N, MN}.

Based on this it can be concluded that a ⊗ (b ⊗ c) = M ⊗ (N ⊗ MN) = {M, MN}≠ (a ⊗ b) ⊗ c = (M ⊗ N) ⊗ MN = {M, N, MN}. So it is not associative. Thus it can be concluded that the algebraic structure that applies to (W, ⊗) is included in the GRUPOID category.

3. Rhesus (Rh) Blood Type System In ABO System Blood Groups

It has been discussed earlier that if the ABO blood type system is associated with the Rhesus system then there are eight probabilities that may occur, including A Rh^+, A Rh^-, B Rh^+, B Rh^-, AB Rh^+, AB Rh^-, O Rh^+ and O Rh^-. If for example the eight blood groups of the ABO system both Rhesus positive and negative we consider as members of a set G then we can write as G = {A Rh^+, A Rh^-, B Rh^+, B Rh^-, AB Rh^+, AB Rh^-, O Rh^+, O Rh^-}. Furthermore, suppose the cross operation of G with itself is (G, ⊗), then the product of the cross operation will be obtained as shown in Table 9.

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Table 9. *Cross Operations of G With It Self*

(G, $\otimes$)	A Rh$^+$	A Rh$^-$	B Rh$^+$	B Rh$^-$	AB Rh$^+$	AB Rh$^-$	O Rh$^+$	O Rh$^-$
A Rh$^+$	A Rh$^+$ O Rh$^+$	A Rh$^+$ A Rh$^-$ O Rh$^+$ O Rh$^-$	A Rh$^+$ B Rh$^+$ AB Rh$^+$ O Rh$^+$	A Rh$^+$ A Rh$^-$ B Rh$^+$ B Rh$^-$ AB Rh$^+$ AB Rh$^-$ O Rh$^+$ O Rh$^-$	A Rh$^+$ B Rh$^+$ ABRh$^+$	A Rh$^+$ A Rh$^-$ B Rh$^+$ B Rh$^-$ AB Rh$^+$ AB Rh$^-$	A Rh$^+$ O Rh$^+$	A Rh$^+$ A Rh$^-$ O Rh$^+$ O Rh$^-$
A Rh$^-$	A Rh$^+$ A Rh$^-$ O Rh$^+$ O Rh$^-$	A Rh$^-$ O Rh$^-$	A Rh$^+$ A Rh$^-$ B Rh$^+$ B Rh$^-$ AB Rh$^+$ AB Rh$^-$ O Rh$^+$ O Rh$^-$	A Rh$^-$ B Rh$^-$ AB Rh$^-$ O Rh$^-$	A Rh$^+$ A Rh$^-$ B Rh$^+$ B Rh$^-$ ABRh$^+$ ABRh$^-$	A Rh$^-$ B Rh$^-$ AB Rh$^-$	A Rh$^+$ A Rh$^-$ O Rh$^+$ O Rh$^-$	A Rh$^-$ O Rh$^-$
B Rh$^+$	A Rh$^+$ B Rh$^+$ AB Rh$^+$ O Rh$^+$	A Rh$^+$ A Rh$^-$ B Rh$^+$ B Rh$^-$ AB Rh$^+$ AB Rh$^-$ O Rh$^+$ O Rh$^-$	B Rh$^+$ O Rh$^+$	B Rh$^+$ B Rh$^-$ O Rh$^+$ O Rh$^-$	A Rh$^+$ B Rh$^+$ ABRh$^+$	A Rh$^+$ A Rh$^-$ B Rh$^+$ B Rh$^-$ AB Rh$^+$ AB Rh$^-$	B Rh$^+$ O Rh$^+$	B Rh$^+$ B Rh$^-$ O Rh$^+$ O Rh$^-$
B Rh$^-$	A Rh$^+$ A Rh$^-$ B Rh$^+$ B Rh$^-$ AB Rh$^+$ AB Rh$^-$ O Rh$^+$ O Rh$^-$	A Rh$^-$ B Rh$^-$ AB Rh$^-$ O Rh$^-$	B Rh$^+$ B Rh$^-$ O Rh$^+$ O Rh$^-$	B Rh$^-$ O Rh$^-$	A Rh$^+$ A Rh$^-$ B Rh$^+$ B Rh$^-$ ABRh$^+$ ABRh$^-$	A Rh$^-$ B Rh$^-$ AB Rh$^-$	B Rh$^+$ B Rh$^-$ O Rh$^+$ O Rh$^-$	B Rh$^-$ O Rh$^-$
AB Rh$^+$	A Rh$^+$ B Rh$^+$ ABRh$^+$	A Rh$^+$ A Rh$^-$ B Rh$^+$ B Rh$^-$	A Rh$^+$ B Rh$^+$ ABRh$^+$	A Rh$^+$ A Rh$^-$ B Rh$^+$ B Rh$^-$	A Rh$^+$ B Rh$^+$ ABRh$^+$	A Rh$^+$ A Rh$^-$ B Rh$^+$ B Rh$^-$	A Rh$^+$ B Rh$^+$	A Rh$^+$ A Rh$^-$ B Rh$^+$ B Rh$^-$

		ABRh$^+$ ABRh$^-$		ABRh$^+$ ABRh$^-$		ABRh$^+$ ABRh$^-$		
AB Rh$^-$	A Rh$^+$ A Rh$^-$ B Rh$^+$ B Rh$^-$ AB Rh$^+$ AB Rh$^-$	A Rh$^-$ B Rh$^-$ AB Rh$^-$	A Rh$^+$ A Rh$^-$ B Rh$^+$ B Rh$^-$ AB Rh$^+$ AB Rh$^-$	A Rh$^-$ B Rh$^-$ ABRh$^-$	A Rh$^+$ A Rh$^-$ B Rh$^+$ B Rh$^-$ ABRh$^+$ ABRh$^-$	A Rh$^-$ B Rh$^-$ AB Rh$^-$	A Rh$^+$ A Rh$^-$ B Rh$^+$ B Rh$^-$	A Rh$^-$ B Rh$^-$
O Rh$^+$	A Rh$^+$ O Rh$^+$	A Rh$^+$ A Rh$^-$ O Rh$^+$ O Rh$^-$	B Rh$^+$ O Rh$^+$	B Rh$^+$ B Rh$^-$ O Rh$^+$ O Rh$^-$	A Rh$^+$ B Rh$^+$	A Rh$^+$ A Rh$^-$ B Rh$^+$ B Rh$^-$	O Rh$^+$	O Rh$^+$ O Rh$^-$
O Rh$^-$	A Rh$^+$ A Rh$^-$ O Rh$^+$ O Rh$^-$	A Rh$^-$ O Rh$^-$	B Rh$^+$ B Rh$^-$ O Rh$^+$ O Rh$^-$	B Rh$^-$ O Rh$^-$	A Rh$^+$ A Rh$^-$ B Rh$^+$ B Rh$^-$	A Rh$^-$ B Rh$^-$	O Rh$^+$ O Rh$^-$	O Rh$^-$

Source: [7]

In Table 9 above, we will study the algebraic structure in (G, $\otimes$). Note that we can see that G on the $\otimes$ operation is a closed operation because all the results of the operation in table 9 above are members of G. To determine whether the $\otimes$ operation on G is associative or not can take any member of G, for example: p = A Rh$^+$, q = A Rh$^-$ and r = O Rh$^+$. note that p $\otimes$ (q $\otimes$ r) = A Rh$^+$ $\otimes$ (A Rh$^-$ $\otimes$ O Rh$^+$) = A Rh$^+$ $\otimes$ (A Rh$^+$, A Rh$^-$, O Rh$^+$, O Rh$^-$) = {(A Rh$^+$ $\otimes$ A Rh$^+$), (A Rh$^+$ $\otimes$ A Rh$^-$), (A Rh$^+$ $\otimes$ O Rh$^+$), (A Rh$^+$ $\otimes$ O Rh$^-$) = {(A Rh$^+$, O Rh$^+$), (A Rh$^+$, A Rh$^-$, O Rh$^+$, O Rh$^-$), (A Rh$^+$, O Rh$^+$), (A Rh$^+$, A Rh$^-$, O Rh$^+$, O Rh$^-$)} = {A Rh$^+$, A Rh$^-$, O Rh$^+$, O Rh$^-$}. further note that (p $\otimes$ q) $\otimes$ r = (A Rh$^+$ $\otimes$ A Rh$^-$) $\otimes$ O Rh$^+$ = (A Rh$^+$, A Rh$^-$, O Rh$^+$, O Rh$^-$) $\otimes$ O Rh$^+$ = {(A Rh$^+$ $\otimes$ O Rh$^+$), (A Rh$^-$ $\otimes$ O Rh$^+$), (O Rh$^+$ $\otimes$ O Rh$^+$), (O Rh$^-$ $\otimes$ O Rh$^+$)} = {(A Rh$^+$, O Rh$^+$), (A Rh$^+$, A Rh$^-$, O Rh$^+$, O Rh$^-$), (O Rh$^+$), (O Rh$^+$, O Rh$^-$)} = {A Rh$^+$, A Rh$^-$, O Rh$^+$, O Rh$^-$}. Take again any member of G for example: k = A Rh$^+$, l = B Rh$^+$ and m = AB Rh$^-$ note that: k $\otimes$ (l $\otimes$ m) = A Rh$^+$ $\otimes$ (B Rh$^+$ $\otimes$ AB Rh$^-$) = A Rh$^+$ $\otimes$ (A Rh$^+$, A Rh$^-$, B Rh$^+$, B Rh$^-$, AB Rh$^+$, AB Rh$^-$) = {(A Rh$^+$ $\otimes$ A Rh$^+$), (A Rh$^+$ $\otimes$ A Rh$^-$), (A Rh$^+$ $\otimes$ B Rh$^+$), (A Rh$^+$ $\otimes$ B Rh$^-$), (A Rh$^+$ $\otimes$ AB Rh$^+$), (A Rh$^+$ $\otimes$ AB Rh$^-$)} = {(A Rh$^+$, O Rh$^+$), (A Rh$^+$, A Rh$^-$, O Rh$^+$, O Rh$^-$), (A Rh$^+$, B Rh$^+$, AB Rh$^+$, O Rh$^+$), (A Rh$^+$, A Rh$^-$, B Rh$^+$, B Rh$^-$, AB Rh$^+$, AB Rh$^-$, O Rh$^+$, O Rh$^-$), (A Rh$^+$, B Rh$^+$, AB Rh$^+$), (A Rh$^+$, A Rh$^-$, B Rh$^+$, B Rh$^-$, AB Rh$^+$, AB Rh$^-$)}= {A Rh$^+$, A Rh$^-$, B Rh$^+$, B Rh$^-$, AB Rh$^+$, AB Rh$^-$, O Rh$^+$, O Rh$^-$}. Now let's look at: (k $\otimes$ l) $\otimes$ m = (A Rh$^+$ $\otimes$ B Rh$^+$) $\otimes$ AB Rh$^-$ = (A Rh$^+$, B Rh$^+$, AB Rh$^+$, O Rh$^+$) $\otimes$ AB Rh$^-$ = {(AB Rh$^-$ $\otimes$ A Rh$^+$), (AB Rh$^-$ $\otimes$ B Rh$^+$), (AB Rh$^-$ $\otimes$ AB Rh$^+$), (AB Rh$^-$ $\otimes$ O Rh$^+$} = {(A Rh$^+$, A Rh$^-$, B Rh$^+$, B Rh$^-$, AB Rh$^+$, AB Rh$^-$), (A Rh$^+$, A Rh$^-$, B Rh$^+$, B Rh$^-$, AB Rh$^+$, AB Rh$^-$), (A Rh$^+$, A Rh$^-$, B Rh$^+$, B Rh$^-$, AB Rh$^+$, AB Rh$^-$), (A Rh$^+$, A Rh$^-$, B Rh$^+$, B Rh$^-$)} = {A Rh$^+$, A Rh$^-$, B Rh$^+$, B Rh$^-$, AB Rh$^+$, AB Rh$^-$}.

It can be seen that in the first part p ⊗ (q ⊗ r) = A Rh$^+$ ⊗ (A Rh$^-$ ⊗ O Rh$^+$) = (p ⊗ q) ⊗ r = (A Rh$^+$ ⊗ A Rh$^-$) ⊗ O Rh$^+$ = {A Rh$^+$, A Rh$^-$, O Rh$^+$, O Rh$^-$}. While in the second part, namely k ⊗ (l ⊗ m) = A Rh$^+$ ⊗ (B Rh$^+$ ⊗ AB Rh$^-$) = {A Rh$^+$, A Rh$^-$, B Rh$^+$, B Rh$^-$, AB Rh$^+$, AB Rh$^-$, O Rh$^+$, O Rh$^-$} ≠ (k ⊗ l) ⊗ m = (A Rh$^+$ ⊗ B Rh$^+$) ⊗ AB Rh$^-$ = {A Rh$^+$, A Rh$^-$, B Rh$^+$, B Rh$^-$, AB Rh$^+$, AB Rh$^-$}. From these two descriptions, we can conclude that the cross operation ⊗ on G is not associative. So we can conclude that (G, ⊗) is a GRUPOID because only the closed property applies.

4. Rhesus (Rh) Blood Type System In MN Blood Type System

The MN blood group system consists of M, N and MN. If the MN blood group system is associated with the rhesus system consisting of positive rhesus (Rh$^+$) and negative rhesus (Rh$^-$) then the blood group consists of M Rh$^+$, M Rh$^-$, N Rh$^+$, N Rh$^-$, MN Rh$^+$ and MN Rh$^-$. Suppose we consider the collection of blood group members as the set H = {M Rh$^+$, M Rh$^-$, N Rh$^+$, N Rh$^-$, MN Rh$^+$, MN Rh$^-$} and if we write the cross operation of H with itself as (H, ⊗) then we will get the cross operation product as shown in Table 10.

Table 10. *Cross Operation of H With Itself*

(H, ⊗)	**M Rh$^+$**	**M Rh$^-$**	**N Rh$^+$**	**N Rh$^-$**	**MN Rh$^+$**	**MN Rh$^-$**
M Rh$^+$	M Rh$^+$	M Rh$^+$ M Rh$^-$	MN Rh$^+$	MN Rh$^+$ MN Rh$^-$	M Rh$^+$ MN Rh$^+$	M Rh$^+$ M Rh$^-$ MN Rh$^+$ MN Rh$^-$
M Rh$^-$	M Rh$^+$ M Rh$^-$	M Rh$^-$	MN Rh$^+$ MN Rh$^-$	MN Rh$^-$	M Rh$^+$ M Rh$^-$ MN Rh$^+$ MN Rh$^-$	M Rh$^-$ MN Rh$^-$
N Rh$^+$	MN Rh$^+$	MN Rh$^+$ MN Rh$^-$	N Rh$^+$	N Rh$^+$ N Rh$^-$	N Rh$^+$ MN Rh$^+$	N Rh$^+$ N Rh$^-$ MN Rh$^+$ MN Rh$^-$
N Rh$^-$	MN Rh$^+$ MN Rh$^-$	MN Rh$^-$	N Rh$^+$ N Rh$^-$	N Rh$^-$	N Rh$^+$ N Rh$^-$ MN Rh$^+$ MN Rh$^-$	N Rh$^-$ MN Rh$^-$
MN Rh$^+$	M Rh$^+$ MN Rh$^+$	M Rh$^+$ M Rh$^-$ MN Rh$^+$ MN Rh$^-$	N Rh$^+$ MN Rh$^+$	N Rh$^+$ N Rh$^-$ MN Rh$^+$ MN Rh$^-$	M Rh$^+$ N Rh$^+$ MN Rh$^+$	M Rh$^+$ M Rh$^-$ N Rh$^+$ N Rh$^-$ MN Rh$^+$ MN Rh$^-$
MN Rh$^-$	M Rh$^+$ M Rh$^-$	M Rh$^-$ MN Rh$^-$	N Rh$^+$	N Rh$^-$	M Rh$^+$ M Rh$^-$	M Rh$^-$ N Rh$^-$

	MN Rh^{+} MN Rh^{-}		N Rh^{-} MN Rh^{+} MN Rh^{-}	MN Rh^{-}	N Rh^{+} N Rh^{-} MN Rh^{+} MN Rh^{-}	MN Rh^{-}

Source: [7]

In table 10 above it is clear that (H, ⊗) is closed. Then we can check whether (H, ⊗) is associative or not by taking any x, y and z members of H, let's say we take x = M Rh^{+}, y = N Rh^{-} dan z = MN Rh^{-}. note that x ⊗ (y ⊗ z) = M Rh^{+} ⊗ (N Rh^{-} ⊗ MN Rh^{-}) = M Rh^{+} ⊗ (N Rh^{-}, MN Rh^{-}) = {(M Rh^{+} ⊗ N Rh^{-}), (M Rh^{+} ⊗ MN Rh^{-})} = {(MN Rh^{+}, MN Rh^{-}), (M Rh^{+}, M Rh^{-}, MN Rh^{+}, MN Rh^{-})} = {M Rh^{+}, M Rh^{-}, MN Rh^{+}, MN Rh^{-}}. Furthermore, note that (x ⊗ y) ⊗ z = (M Rh^{+} ⊗ N Rh^{-}) ⊗ MN Rh^{-} = (MN Rh^{+}, MN Rh^{-}) ⊗ MN Rh^{-} = {(MN Rh^{+} ⊗ MN Rh^{-}), (MN Rh^{-} ⊗ MN Rh^{-})} = {(M Rh^{+}, M Rh^{-}, N Rh^{+}, N Rh^{-}, MN Rh^{+}, MN Rh^{-}), (M Rh^{-}, N Rh^{-}, MN Rh^{-})} = {M Rh^{+}, M Rh^{-}, N Rh^{+}, N Rh^{-}, MN Rh^{+}, MN Rh^{-}}. From these two descriptions it is clear that: x ⊗ (y ⊗ z) = M Rh^{+} ⊗ (N Rh^{-} ⊗ MN Rh^{-}) = {M Rh^{+}, M Rh^{-}, MN Rh^{+}, MN Rh^{-}} ≠ (x ⊗ y) ⊗ z = (M Rh^{+} ⊗ N Rh^{-}) ⊗ MN Rh^{-} = {M Rh^{+}, M Rh^{-}, N Rh^{+}, N Rh^{-}, MN Rh^{+}, MN Rh^{-}}. It can be concluded that (H, ⊗) is not associative. Therefore (H, ⊗) is a GRUPOID.

IV. CONCLUSIONS

From the above study, it can be concluded that the algebraic structure of the product of the cross operation between the ABO blood group system and itself includes an algebraic structure of the groupoid type. The cross operation between the MN blood group system and itself also includes an algebraic structure of the groupoid type. The cross operation between the ABO blood group system and itself in the rhesus system is a groupoid. And the cross operation between the MN blood group system and itself in the rhesus system is also a groupoid.

For the future, researchers hope that there will be similar research that discusses the algebraic structure that exists in non-main blood group systems such as Lutheran, Kell, Lewis, Duffy and so on. Whether it is a cross operation between certain blood groups and themselves or a cross operation if it is associated with a positive or negative rhesus system. Thus, it is hoped that this research can develop into a broader and more complex realm.

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