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ROBUST PREDICTION INTERVALS FOR INDONESIAN INFLATION: A BIAS-CORRECTED BOOTSTRAP APPROACH

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Abstract. Inflation is important to be analyzed due to its impact is felt across various aspects of the economy and individuals' lives. This research aimed to develop robust and reliable predictions concerning Indonesian inflation using the bias-corrected bootstrap method for an AR model. The data utilized spanned from January 2020 to September 2023 and was obtained from Bank Indonesia's website. The analysis provided the optimal order in the AR model, which resulted in p=2 as the best order (AIC=-1.858, BIC=-1.698, and HQ=-1.798). The number of bootstrap replications used was B=100, 250, 500, and 1000. The analysis was conducted using R Studio. The analysis results indicated that the model employed for prediction analysis was highly stable, with all point forecasts indicating result consistency. The prediction results suggested that inflation in Indonesia was expected to decrease in the upcoming 5 months. The results also revealed that the bias-corrected bootstrap approach could provide forecasting results with a higher level of accuracy. This research contributed to the understanding and forecasting of Indonesian inflation, emphasizing model stability and consistent results.

Keywords: inflation, AR model, bias-corrected bootstrap, prediction

I. PENDAHULUAN

Inflation in Indonesia refers to the sustained increase in the general prices of goods and services over a specified period [1]. It is a significant economic indicator that impacts consumer purchasing power, economic stability, and monetary policy decisions made by Bank Indonesia, the country's central bank. Indonesia experiences fluctuating inflation rates over time means that the country has observed changes in its inflation rate over different periods, and these rates have not remained constant. Inflation rates in Indonesia have shown variation in response to various economic and external factors [2].

During the period from January 2020 to September 2023, the inflation condition in Indonesia experienced various fluctuations and changes due to various economic and non-economic factors. At the beginning of 2020, inflation in Indonesia was within a controlled range. The government and the Central Bank of Indonesia (Bank Indonesia or BI) worked to maintain inflation within the set target. With the emergence of the COVID-19 pandemic in early 2020, the Indonesian economy faced significant pressures [3], [4]. Lockdowns and economic activity restrictions had an impact on inflation [5], [6], with some commodities experiencing price declines [7]. In 2021, there was an economic recovery as COVID-19 vaccinations became widespread, leading to price increases in certain commodities and services. Throughout this period, there were fluctuations in specific prices, such as food and energy prices, which could influence overall inflation rates. The inflation situation in Indonesia can continue to change in response to global economic events and changes. Factors such as global crude oil prices, changes in monetary policies, and domestic economic conditions will continue to affect inflation rates in the future.

Overall, the inflation conditions in Indonesia during this period reflect the challenges and changes faced by the national economy due to the COVID-19 pandemic and global dynamics. Bank Indonesia and the government continue to work to maintain inflation stability while supporting sustainable economic growth [8]. However, fluctuations in inflation are common in economies, and they are influenced by a complex interplay of domestic and global factors, government policies, and economic events.

Inflation forecasting is a critical task in economics, influencing monetary policy decisions, financial markets, and overall economic stability. Accurate and reliable inflation predictions are essential for policymakers and investors alike. In the context of Indonesia, an emerging economy with unique economic and financial dynamics, predicting inflation is a complex challenge due to various factors such as changing market conditions, policy interventions, and global economic influences.

Traditional time series forecasting techniques, including the Autoregressive model, have been widely used for inflation prediction. However, these models often assume Gaussian distribution and constant error variance, which may not capture the true characteristics of true data [9]–[11]. Additionally, standard forecasting methods may not adequately account for biases or provide robust prediction intervals [9], [12]. To navigate these challenges effectively, robust and innovative forecasting methods, such as the bias-corrected forecasting and bootstrap prediction intervals approach, can offer valuable insights and contribute to the country's economic stability and growth.

The novelty of this research lies in the application of a bias-corrected forecasting and bootstrap prediction intervals approach to Indonesian inflation forecasting. Traditional forecasting methods may introduce bias, especially in the presence of outliers or non-Gaussian data distributions [13]–[16]. The proposed approach addresses this issue by implementing bias correction techniques, ensuring that predictions are more accurate and unbiased. Further, robust prediction intervals are crucial for quantifying uncertainty in inflation forecasts. The bootstrap method allows for the generation of prediction intervals that capture the variability and potential outliers in the data [15]–[17], providing more reliable estimates of future inflation rates. The bias-corrected forecasting and bootstrap approach can accommodate various data distributions and nonlinear relationships, making it adaptable to the specific characteristics of Indonesian inflation data.

By applying this method to Indonesian inflation forecasting, this research aims to enhance the accuracy and reliability of inflation predictions, thereby aiding policymakers, financial institutions, and investors in making informed decisions. Moreover, this research contributes to the broader field of time series forecasting by demonstrating the applicability of bias correction and robust prediction intervals in a real-world economic context.

II. METHODOLOGY

2.1. Data

This study utilizes secondary data from the inflation dataset in Indonesia, publicly available through Bank Indonesia's website (https://www.bi.go.id/en/statistik/indikator/data-inflasi.aspx). The time series data for Indonesian inflation analyzed in this research spans from January 2020 to September 2023, encompassing a total of 45 data series. Figure 1 below provides a visual representation in the form of a time series plot for this dataset.

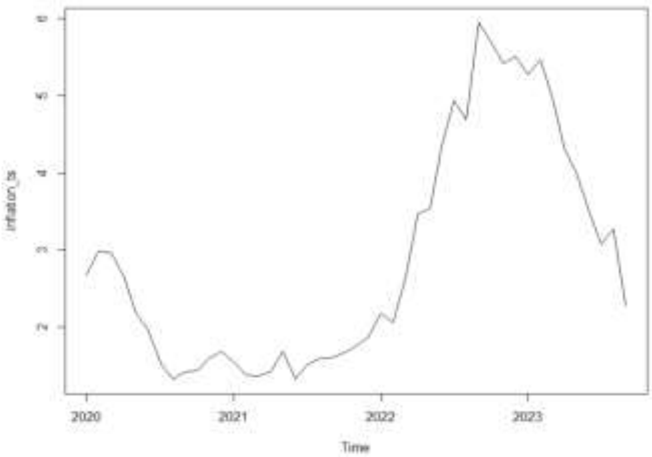


Figure 1. Indonesia Inflation Data (2020-2023)

The figure above illustrates that Indonesia has experienced fluctuations in inflation during the period from January 2020 to September 2023. The highest inflation occurred in September 2022, reaching 5.95%. In contrast, the lowest inflation was recorded in August 2020, at 1.32%. This data reveals the dynamic nature of inflation in Indonesia over the specified timeframe. Inflation rates have not remained static but have rather exhibited variations.

2.2. Autoregressive (AR) Model

The AR model assumes that each data point in the series is a linear combination of its previous values, often referred to as lags [18]–[21]. In essence, it captures the temporal dependencies within the data, making it a valuable tool for understanding and forecasting time-dependent phenomena.

Suppose an autoregressive model is defined as AR(p), which incorporates a time-varying deterministic component, denoted as $D_{j,t}$. This component encompasses intercepts, time trends, and dummy variables. The AR model can be expressed as follows [12], [17]:

$$Y_t = \sum_{i=1}^{p} \gamma_i Y_{t-1} + \sum_{j=1}^{m} \beta_j D_{j,t} + u_t \tag{1}$$

Where u_t represents the error term. Let's consider W as an nxp matrix, which serves as a matrix of lagged dependent variables, and define D as an nxm matrix, representing the matrix of deterministic components. Then $Z = [W: D]$ is a matrix of size nxk where $k = p + m$. Consider $Y = (Y_1, Y_2, \dots, Y_n)'$, $u = (u_1, u_2, \dots, u_n)'$ then the equation (1) can be written as follows:

$$Y = Z\alpha + u \tag{2}$$

If the vector of unknown coefficients is defined as $\alpha = (\gamma: \beta)'$ then the least-squares estimator of α and σ is defined as follows:

$$\hat{\alpha} = (\hat{\gamma}, \hat{\beta}) = \frac{Z'Y}{Z'Z} \tag{3}$$
$$s^2 = \frac{e'e}{(n-k)}$$

where $e = (e_1, e_2, \dots, e_n)'$ are the residuals. Let the optimal forecast be defined as follows:

$$Y_{n+h} = Y_n(h) + u_{n+h} \tag{4}$$

where

$$Y_n(h) = \sum_{i=1}^{p} \gamma_i Y_n(h-i) + \sum_{j=1}^{m} \beta_j D_{j,n+h} \tag{5}$$

For $h \leq 0$, $Y_n(h)$ can be estimated as follows:

$$\hat{Y}_n(h) = \sum_{i=1}^{p} \hat{\gamma}_i Y_n(h-i) + \sum_{j=1}^{m} \hat{\beta}_j D_{j,n+h} \tag{6}$$

From equation (5), $\hat{Y}_n(h)$ is asymptotically consistent and normally distributed [12]. By utilizing the normal approximation, asymptotic prediction intervals can be constructed. Unfortunately, conventional forecasting techniques might not sufficiently address biases or offer resilient prediction intervals. To effectively address these hurdles, resilient and groundbreaking forecasting techniques, such as the approach involving bias-corrected forecasting and bootstrap prediction intervals, are needed [22], [23].

2.3. Bias-corrected estimators for AR models

Based on equation (1), the nonparametric bootstrap approach using residual resampling can be implemented to estimate the bias of $\hat{\alpha}$ in $O(n^{-1})$. Let's assume the bootstrap sample is defined as $\{Y_t^*\}_{t=1}^{n}$, which is degenerated by using the initial data points $\{Y_t\}_{t=1}^{p}$ as follows [12], [22]:

$$Y_t^* = \sum_{i=1}^{p} \hat{\gamma}_i Y_{t-i}^* + \sum_{j=1}^{m} \hat{\beta}_j D_{j,t} + e_t^* \tag{7}$$

Where e_t^* represents random samples drawn with replacement based on $\{e_t\}_{t=1}^n$. Then, the bootstrap estimator for α, is denoted as $\hat{\alpha}^* = \frac{Z^*Y^*}{Z^{*\prime}Z^*}$. The bootstrap bias-corrected estimator is obtained from:

$$\begin{gathered} \hat{\alpha}_B^C = \hat{\alpha} - bias(\hat{\alpha}) \\ \hat{\alpha}_B^C = \left[\hat{\gamma}_B^C : \hat{\beta}_B^C\right] \end{gathered} \tag{8}$$

By utilizing equation (8), here are the steps for bootstrap bias-correction estimation:

Step 1: Calculate the estimator $\hat{\alpha}$ and s^2 for the model in equation (1) and compute the bias-corrected estimator based on equation (8). Let the residuals from the computation of $\hat{\alpha}$ be defined as $\{e_t^C\}_{t=1}^n$.

Step 2: Using the initial data points $\{Y_t\}_{t=1}^p$, generate the bootstrap sample $\{Y_t^*\}_{t=1}^n$ as $Y_t^* = \sum_{i=1}^p \hat{\gamma}_i^C Y_{t-i}^* + \sum_{j=1}^m \hat{\beta}_j^C D_{j,t} + e_t^*$. The sample e_t^* is randomly drawn with replacement. Using the bias estimate from step 1, the bias correction is defined as $\hat{\alpha}^{C*} = \hat{\alpha}^* - bias(\hat{\alpha})$

Step 3: Repeat step 2 for B times to generate the bootstrap distribution for the forecast, $\{Y_n^{C*}(h;j)\}_{j=1}^B$.

Then, the nominal coverage rate of 100(1-θ)% for the bootstrap bias-corrected prediction interval is provided by:

$$[Y_n^*(h,\tau), Y_n^*(h,1-\tau)] \tag{9}$$

Where $Y_n^*(h,\tau)$ is the 100τ-th percentile of the bootstrap distribution $\{Y_n^{C*}(h;i)\}_{i=1}^B$, and $\tau = 0.5\theta$.

III. CASE STUDY

To obtain point forecasts and prediction intervals, this research employed bias-correction methods in autoregressive time series models. These methods are crucial because point forecasts alone do not provide a complete picture. Without prediction intervals, it is challenging to assess the accuracy and uncertainty of the forecasts [24]–[26].

In the AR(p) model, it is crucial to determine the appropriate AR order using statistical criteria such as the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), or Hannan-Quinn Information Criterion (HQ). These criteria assist in selecting the optimal order (p) for the Autoregressive (AR) model. Models with the lowest AIC, BIC, or HQ values are considered the most suitable. Lower values indicate a better balance between model fit quality and complexity. The analysis results indicated that the lowest AIC, BIC, and HQ values occurred at order p=2 (AIC=-1.858, BIC=-1.698, and HQ=-1.798).

To obtain bias-corrected forecasting and bootstrap prediction interval estimates for Indonesian inflation data, this research employed various numbers of bootstrap replications (B=100, 250, 500, and 1000). The prediction intervals utilized were 5% for the lower bound and 95% for the upper bound. The prediction analysis was conducted for the upcoming 5 periods, which correspond to 5 months following September 2023. The following table illustrates the results of the prediction analysis for Indonesian inflation data, using time series data from January 2020 to September 2023.

Table 1. Forecasting analysis results

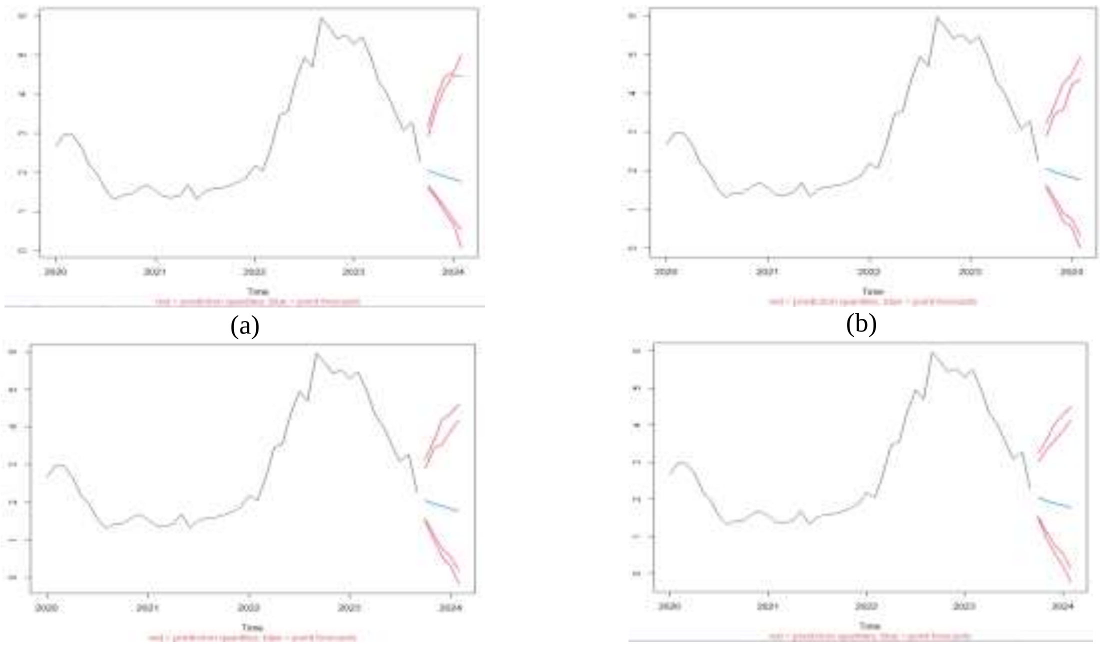
Periods	B=100			B=250			B=500			B=1000		
	point forecast	5%	95%	point forecast	5%	95%	point forecast	5%	95%	point forecast	5%	95%
$h1$	2.04	1.43	2.91	2.04	1.50	2.94	2.04	1.48	2.98	2.04	1.47	3.10
$h2$	1.94	0.93	2.99	1.94	1.03	3.32	1.94	1.05	3.43	1.94	1.00	3.48
$h3$	1.87	0.59	3.26	1.87	0.74	3.66	1.87	0.75	3.67	1.87	0.74	3.77
$h4$	1.80	0.38	3.43	1.80	0.50	3.73	1.80	0.64	3.82	1.80	0.52	4.06
$h5$	1.73	0.15	3.59	1.73	0.20	4.07	1.73	0.25	4.00	1.73	0.26	4.20

From Table 1, the values of 5% and 95% referred to prediction intervals that encompass 90% of the prediction distribution, with 5% on the lower side (lower bound) and 95% on the upper side (upper bound). The results of the prediction analysis using bias-corrected forecasts and bootstrap prediction intervals in Table 1 indicated consistent figures, with no significant differences observed. All point forecasts generated from varying numbers of bootstrap replications indicated the consistency of the results [24]. Table 1 also demonstrated that the model employed for prediction analysis was highly stable and not sensitive to variations in bootstrap data, as its point forecasts remained relatively consistent regardless of the value of B used.

By examining the trend from the prediction results in Table 1, it could be concluded that inflation in Indonesia was expected to decrease in the upcoming 5 months. The prediction values were progressively lower as time passed. This might serve as an indication for analysts or forecasting models. It's important to remember that these results provided a general overview of the expected direction of inflation movement, and there was a possibility of unexpected variability or changes in economic conditions that could influence actual outcomes. Therefore, prediction intervals were always necessary to accompany point forecasts, as they encompassed the level of uncertainty.

Furthermore, plotting prediction intervals and point forecasts was important. Plotting these intervals alongside point forecasts, visually represented the level of uncertainty associated with your predictions. This helped decision-makers and stakeholders understand the potential variability in future outcomes.

Figure 2 below displays the plotting of prediction intervals and point forecasts using varying numbers of bootstrap replications, namely B=100, 250, 500, and 1000. This research employed quantile estimates, specifically, quartiles from the estimated distribution. Prediction intervals were utilized to examine the specified coverage probability range under the distribution [24], [25]. In this research, a 95% prediction interval was employed, determined by the 2.5% and 97.5% quantiles of the forecast distribution. This 95% prediction interval was one of the commonly used ones in forecasting analysis, alongside the 80% prediction interval [24], [26], [27].



(a) (b) (c) (d)

Figure 2. a) Time plot and prediction intervals (B=100); b) Time plot and prediction intervals (B=250); c) Time plot and prediction intervals (B=500); d) Time plot and prediction intervals (B=1000)

From the figure above, the blue line represents the point forecasts for the predicted data over the next 5 periods (months). Meanwhile, the red line represents the prediction intervals. It should be noted that when the prediction analysis indicated a higher level of uncertainty, the prediction intervals tended to widen. Plotting prediction intervals could also help evaluate the performance of forecasting models. From Figure 2, it could be seen that the point forecasts consistently fall within the prediction intervals; it suggested that the model was capturing the variability in the data effectively.

Furthermore, from Figure 2, it was also evident that the prediction intervals for the number of bootstrap replications B=100 and B=250 appeared less smooth. On the other hand, for B=500 and B=1000, they appeared much smoother. Increasing the number of bootstrap replications resulted in smoother prediction intervals. Additionally, from Figure 2, it could be observed that as the number of bootstrap iterations used increased, the prediction intervals appeared narrower. Therefore, it could be stated that the bias-corrected bootstrap approach could provide forecasting results with a higher level of accuracy.

The larger the number of bootstrap replications, B, the closer it approached the true distribution of the data, and the bootstrap results became more stable. The figure above illustrates that the analysis results with higher B values yielded narrower 5% and 95% prediction intervals (closer together) compared to lower B values. This indicated that with an increase in B, the uncertainty in your predictions had reduced. In other words, the model being used was more confident in making more accurate predictions [10], [24].

IV. CONCLUSION

In conclusion, this research has yielded several key findings. Firstly, the analysis revealed that the chosen model for prediction analysis demonstrates a high level of stability. It remains robust and unaffected by variations in bootstrap data, with point forecasts consistently maintaining relative consistency across various values of B. Secondly, based on the prediction results; it can confidently assert that inflation in Indonesia is anticipated to exhibit a decreasing trend over the next 5 months. The values of the predictions progressively decline as time advances, implying a potential economic trend. Furthermore, the analysis consistently shows that the point forecasts consistently align within the prediction intervals. This suggests that the proposed model effectively captures the underlying variability in the data, enhancing the reliability of the forecasts. As the number of bootstrap iterations increases, the prediction intervals become narrower and smoother, further reinforcing the robustness of our approach. In summary, this research presents a robust and reliable framework for forecasting Indonesian inflation, providing valuable insights into future economic trends while emphasizing the stability and accuracy of the model.

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