**FROZEN INITIAL LIABILITY METHOD TO DETERMINE NORMAL COST OF PENSION FUND WITH VASICEK INTEREST RATE MODEL**

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**Abstract.** Civil servants have an important role in national development, so increasing their productivity is needed. The pension fund program is given as a form of effort by government agencies to ensure employee welfare when entering retirement. This research discusses the normal cost of the defined benefit pension program using one of the actuarial valuation methods, namely Frozen Initial Liability (FIL), by taking into account the stochastic interest rate following the Vasicek model. The data used in this study are lecturers majoring in MIPA, Faculty of Science and Technology, Universitas Jambi, consisting of 8 people of female gender with the status of being a participant since 2022. Based on the calculation results obtained that in the period 0-30 years, the normal cost for each group member is constant, namely Rp4,144.278. per year or Rp342.856, per month. When the working period entered 31 years, one by one the participants began to enter their retirement period, which resulted in a change in the normal cost value. At 38 years of service, there was only one participant with a normal cost of Rp3,065.294, per year or by Rp255,441, per month. Changes in normal cost tend to decrease when retirement program participants also decrease. In the period of more than 38 years, all participants have retired so that normal cost payments are stopped.

**Keywords:** Frozen Initial Liability, Normal Cost, Vasicek

I. INTRODUCTION

Pension funds are legal entities that manage and run programs that promise retirement benefits for workers, as a form of reward for contributions that have been made during the working period. Pension programs are divided into two types, namely defined benefit pension programs and defined contribution pension programs. If the amount of pension benefits is determined in accordance with pension fund regulations based on actuarial calculation methods, it is called a defined benefit pension program. Meanwhile, a defined contribution pension plan is a program whose contributions have been determined and invested in financial assets and then recorded in each participant’s account as a retirement benefit [1].

There are several actuarial calculation methods used to determine pension fund projections in the form of normal costs and actuarial liabilities. In addition to the selection of methods, the interest rate is also important because it always change over time. There are two categories in actuarial calculation, namely the Projected Benefit Cost Method and the Accrued Benefit Cost Method. One example of the Projected Benefit Cost Method category is the Entry Age Normal method, while the Projected Unit Credit method is included in the Accrued Benefit Cost Method.

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Method category [2]. Some researchers have examined the comparison of the Projected Unit Credit and Entry Age Normal methods, in general, it is found that the normal cost of constant interest rates is greater than the Vasicek interest rate [3], [4]. In research [5] using the Entry Age Normal method by comparing constant interest rates and Vasicek obtained that the normal cost and liability of constant interest rate are greater. If the entry age work increases, the normal cost also increases, but on the other hand the actuarial liability decreases.

The Entry Age Normal method considers the age when registered as a participant until reaching retirement age, a derivative of this method is called the Frozen Initial Liability (FIL) method. The Frozen Initial Liability method is used in calculating pension fund in a particular group or group by assuming that the amount of contributions to be paid by each member is the same even though they have different retirement benefits [6].

Several researchers have studied the determination of normal cost and actuarial liabilities using the Entry Age Normal and Frozen Initial Liability methods with constant interest rates. In research [7] obtained that the normal cost obtained using the Frozen Initial Liability method will decrease if there are group members who enter normal retirement age while actuarial liabilities will continue to increase. Based on the research results [8] obtained that the normal cost of the Entry Age Normal method is constant every year but different for each participant because it depends on the length of service, the Frozen Initial Liability method decreases as active members decrease and is the same for each participant.

Based on these conditions, the use of stochastic interest rates is interesting to study. This research uses the Frozen Initial Liability method by using the Vasicek interest rate which is stochastic. The Vasicek interest rate model predicts the movement of interest rates for the next time by looking at the previous movements and always moving towards an equilibrium point [9].

II. METHODOLOGY

2.1 Survival Function

A person can retire due to disability, death, resignation, and when reaching normal retirement age. The survival function referred to in this study is the chance that someone is still actively working to reach normal retirement age by referring to the 2019 Indonesian Mortality Table. The chance of an employee aged \( x \) years old continues to work until the age of \( x + t \) year is expressed by [10]:

\[
\begin{align*}
tp_x &= \frac{l_{x+t}}{l_x} \\
\end{align*}
\]

with

\( tp_x \): The probability that a person aged \( x \) years old can work until the age of \( x + t \) years

\( l_{x+t} \): Number of people exactly aged \( x + t \) years

\( l_x \): Number of people exactly aged \( x \) years

2.2 Vasicek Interest Rate Model

The Vasicek interest rate model is a stochastic model that has a tendency to return to an equilibrium point after experiencing a decrease or increase, expressed as follows [4]:

\[
dr(t) = \alpha(\mu - r(t))dt + \sigma dW(t)
\]

with
\[ \alpha = \text{Speed of interest rate adjustment towards the long-term interest rate level} \]
\[ \mu = \text{Long-term average interest rate} \]
\[ \sigma = \text{Interest rate volatility} \]
\[ r(t) = \text{Interest rate at time } t \]
\[ W(t) = \text{Wiener process at time } t. \]

The value of the Vasicek model interest rate parameter is calculated using Bank Indonesia interest rate data for 2018-2022, obtained using the following equation [11]:

\[
\alpha = \frac{n^2 - 2n + 1 + \sum_{i=1}^{n-1} r_{t+1} \sum_{i=1}^{n-1} \frac{1}{r_t} - \sum_{i=1}^{n-1} r_t \sum_{i=1}^{n-1} \frac{1}{r_t} - (n - 1) \sum_{i=1}^{n-1} \frac{r_{t+1}}{r_t} \Delta t}{n^2 - 2n + 1 - \sum_{i=1}^{n-1} r_t \sum_{i=1}^{n-1} \frac{1}{r_t}} \tag{3}
\]

\[
\mu = \frac{(n - 1) \sum_{i=1}^{n-1} r_{t+1} - \sum_{i=1}^{n-1} \frac{r_{t+1}}{r_t} \sum_{i=1}^{n-1} r_t}{(n^2 - 2n + 1 + \sum_{i=1}^{n-1} r_{t+1} \sum_{i=1}^{n-1} \frac{1}{r_t} - \sum_{i=1}^{n-1} r_t \sum_{i=1}^{n-1} \frac{1}{r_t} - (n - 1) \sum_{i=1}^{n-1} \frac{r_{t+1}}{r_t})} \tag{4}
\]

\[
\sigma = \sqrt{\frac{1}{n - 2} \sum_{i=1}^{n-1} \left( \frac{r_{t+1} - r_t}{\sqrt{r_t}} - \frac{\mu}{\sqrt{r_t}} + \alpha \sqrt{r_t} \right)^2}. \tag{5}
\]

### 2.3 Interest Rate Function

The interest rate used to determine the deductibility of future payments is also referred to as the present value. For a constant interest rate, if \( i \) denotes the interest rate in year \( t \) with \( t = 1, 2, ..., n \) then the present value of a unit of money in \( n \) year is expressed by [12]:

\[
v^n = \frac{1}{(1 + i)^n}. \tag{6}
\]

This study uses a stochastic interest rate model, so the form of \( v^n \) in equation (6) is replaced with \( P(t) \) which states the expected cash value of payment by one unit following the Vasicek model interest rate [4]:

\[
P(t) = \exp \left( (B(t) - t) \left( \mu - \frac{\sigma^2}{2\alpha^2} \right) - \frac{\sigma^2 B(t)^2}{4\alpha} - r(0) B(t) \right) \tag{7}
\]

with \( r(0) = r_0 \) and \( B(t) = \frac{1 - \exp(-\alpha t)}{\alpha} \).

### 2.4 Life Annuity Function

A life annuity is a series of payments that a person make as long as the person is alive. The notation \( \ddot{a}_x \) is the initial life annuity for age \( x \) years, and \( \ddot{a}_{x:n} \) is the present value of the initial annuity of \( n \) years, expressed as follows [13]:

\[
\ddot{a}_x = \frac{N_x}{D_x} \tag{8}
\]

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\[ \dot{a}_{x:n} = \frac{N_x - N_{x+n}}{D_x}, \]  

(9)

Notation \( N_x \) dan \( D_x \) are commutation symbols simplifying calculations in calculating mortality tables, expressed by [2]:

\[ D_x = v^x l_x, \]  

(10)

\[ N_x = D_x + D_{x+1} + D_{x+2} + \cdots + D_\omega = \sum_{i=0}^{\omega-x} D_{x+i}, \]  

(11)

in this study, \( \omega \) is the highest age in the 2019 Indonesian Mortality Table.

### 2.5 Benefit Function

The benefit function is used to determine the amount of benefits that will be received by participants when they reach retirement time. The formula for determining retirement benefits is [14]:

(a) **Last Salary**

The amount of retirement benefits at age \( r \) year is

\[ B_r = k(r - y)s_{r-1} \]  

(12)

with \( B_r \) denotes the size of the retirement benefit, \( k \) is the proportion of salary prepared for retirement benefits that will be received upon reaching retirement age, \( y \) is the age when becoming a pension fund participant, \( r \) is the normal retirement age, \( r - y \) is the length of service, and \( s_{r-1} \) is the last year’s salary before retirement.

(b) **Average Last Salary**

if \( s_t \) denotes the salary in year \(-t\), then the amount of retirement benefits at age \( r \) year is

\[ B_r = k(r - y)\frac{1}{n} \sum_{t=r-n}^{r-1} s_t \]  

(13)

(c) **Average Salary During \( n \) Year of Employment**

The amount of retirement benefits at age \( r \) year is

\[ B_r = kS_r \]  

(14)

with \( k \) is the percentage of salary given and \( S_r \) is the total salary during employment.

The value of \( S_r \) is calculated using the equation [2]:

\[ S_r = \sum_{t=y}^{r-1} s_t. \]  

(15)

### 2.6 Normal Cost (NC)

Normal Cost (NC) is a payment made by the participant every year to the pension fund since entering work, aiming to install the FVFB of each participant. In principle, the Frozen Initial Liability (FIL) method functions to calculate the aggregate normal cost for one cohort, in contrast to the EAN method which calculates the normal cost of each participant. The normal cost

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cost value in this method will decrease if there are participants who leave the pension program. The Frozen Initial Liability (FIL) method used the number of participants in a group, so the formula for calculating normal cost is stated as follows [8]:

\[
FIL(NC)_{tj} = \frac{1}{m_t} \sum_{j \in A_t} B_{rj} \bar{a}_r \frac{D_{rj}}{N_{y_j} - N_{r_j}}
\]  

with

- \(FIL(NC)_{tj}\) : Normal Cost paid by the individual \(j\) at the time of \(t\) year
- \(j\) : Individuals who are pensionable members of the set of members \(A_t\)
- \(D_{rj}\) : Commutation symbol for individuals \(j\) which expresses the result of multiplying the discount factor by the number of people exactly at the age of \(r\) year
- \(N_{y_j}\) : The sum of \(D_{yj}\) which starts from the age of \(y\) years to the highest age reached in the Mortality Table
- \(N_{r_j}\) : The sum of \(D_{rj}\) which starts from the age of \(r\) years to the highest age reached in the Mortality Table
- \(A_t\) : The set of retired members at time \(t\)
- \(B_{rj}\) : Pension benefits for individuals \(j\) who retires at age \(r\) years
- \(m_t\) : Number of employees participating in the pension program at the time \(t\) year.

### III. RESULTS AND DISCUSSION

The data used in this study are primary data of lecturers majoring in MIPA Faculty of Science and Technology consisting of 8 people of female gender with civil servant status in 2022. Based on the data obtained, the assumptions used in this study are as follows:

1. Mortality rates are assumed to follow the 2019 Indonesian Mortality Table (Female)
2. All participants retire normally at age 65 (\(r = 65\))
3. The graduate status of all participants is S2, the civil servant class at the beginning of the working year is IIIb, and the basic salary at the beginning of the working year is Rp2,688,500,-
4. The age of all participants at the beginning of the working year are: the first participant was 26 years old, the second 27 years old, the third 28 years old, the fourth 30 years old, the fifth 31 years old, the sixth 32 years old, the seventh 32 years old, and the eighth 34 years old.
5. Civil servant class increases are influenced by length of service carried out every 5 years while other government policies are considered non-existent.
6. The salaries of pension fund participants are based on the civil servant salary scale table contained in a copy of Government Regulation No. 15/2019.
7. The percentage of a participant’s salary that is set aside for retirement benefits is 2,5\%.
8. The interest rate follows the Vasicek model, the value of which can differ every year.

#### 3.1 Retirement Benefits

In the previous assumption, it has been explained that the data used in this study consists of 8 people, therefore, first determine the amount of benefits that will be received by each group member when they retire. In Table 1, information is given about the participant data used along with the total salary of each participant during work.
To calculate the value of \( A \) life annuity, a series of payments that a person makes as long as the person is alive. To calculate the value of \( D_r \) and \( N_r \) first determine the expected cash value of the payment by one unit using the parameter value that follows the Vasicek model. In this study, the parameter values \( \alpha \), \( \mu \), and \( \sigma \) in equations (3)-(5) are calculated based on monthly data of Bank Indonesia interest rates from January 2018-December 2022, so the results obtained are as follows \( \alpha = 0.008196205 \), \( \mu = 0.070322289 \), and \( \sigma = 0.131469885 \). To calculate the value of \( P(t) \) in equation (7) is used the value of \( r_0 = 0.055 \) which is the value of the Bank Indonesia interest rate in December 2022. The complete expectation of the cash value of payment by one unit is given in Figure 1 below:

### 3.2 Life Annuity Function
Values $P(t)$ in Figure 1 are used in the calculation of the values $D_r$ dan $N_r$ to obtain the initial lifetime annuity value. Notation $\bar{a}_r$ is the initial lifetime annuity for retirement age $r$ years, the following results are obtained:

$$\bar{a}_{65} = \frac{N_{65}}{D_{65}} = 11,62142959.$$ 

### 3.3 Normal Cost (NC)

The Frozen Initial Liability (FIL) method is a derivative of the Entry Age Normal method. This method is used to calculate the value of payments made by participants in a certain group to the pension fund since entering work. This method assumes that the amount of contributions to be paid by each group member is the same every year. This section describes several cases that cause changes in normal cost when a group member leaves the pension funding systems, when entering normal retirement age. The complete normal cost calculation for each event is described as follows:

1. When $t = 0$ to $t = 30$.

In the 0-30 year period, the pension plan group consists of 8 members ($m_t = 8$). Using equation (16), the normal cost that will be paid by each group member is:

$$FIL(\text{NC})_{t_j} = \frac{1}{m_t} \sum_{j \in A_t} B_{r_j} \bar{a}_r \frac{D_{r_j}}{N_{y_j} - N_{r_j}}$$

$$FIL(\text{NC})_{t_j} = \frac{1}{8} \left( B_{651} \bar{a}_{65} \frac{D_{651}}{N_{261} - N_{651}} + B_{652} \bar{a}_{65} \frac{D_{652}}{N_{272} - N_{652}} + B_{653} \bar{a}_{65} \frac{D_{653}}{N_{283} - N_{653}} 
+ B_{654} \bar{a}_{65} \frac{D_{654}}{N_{304} - N_{654}} + B_{655} \bar{a}_{65} \frac{D_{655}}{N_{315} - N_{655}} + B_{656} \bar{a}_{65} \frac{D_{656}}{N_{326} - N_{656}} 
+ B_{657} \bar{a}_{65} \frac{D_{657}}{N_{327} - N_{657}} + B_{658} \bar{a}_{65} \frac{D_{658}}{N_{386} - N_{658}} \right)$$

$FIL(\text{NC})_{t_j} = Rp4.114.278, -$.

So, the amount of normal cost that must be paid by each participant is the same, which is equal to $Rp4.114.278, -$ per year or by $Rp342.856, -$ per month.

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**Gambar 1. Expected Cash Value of Payments with Interest Rate Vasicek Model**

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2. When $t = 31$ to $t = 32$.
At 31-32 years, the group pension plan consists of 7 members ($m_t = 7$). The normal cost that will be paid by each group member is:

$$FIL(NC)_{t_j} = \frac{1}{7} \left( B_{65_1} \ddot{a}_{65} \frac{D_{65_1}}{N_{26_1} - N_{65_1}} + B_{65_2} \ddot{a}_{65} \frac{D_{65_2}}{N_{27_2} - N_{65_2}} + B_{65_3} \ddot{a}_{65} \frac{D_{65_3}}{N_{28_3} - N_{65_3}} + B_{65_4} \ddot{a}_{65} \frac{D_{65_4}}{N_{30_4} - N_{65_4}} + B_{65_5} \ddot{a}_{65} \frac{D_{65_5}}{N_{31_5} - N_{65_5}} + B_{65_6} \ddot{a}_{65} \frac{D_{65_6}}{N_{32_6} - N_{65_6}} + B_{65_7} \ddot{a}_{65} \frac{D_{65_7}}{N_{32_7} - N_{65_7}} \right).$$

$$FIL(NC)_{t_j} = Rp3.933.682, -.$$

So, the amount of normal cost that must be paid by each participant is the same, which is equal to Rp3.933.682, – per year or by Rp327.807, – per month.

3. When $t = 33$
When the service period reaches 33 years, the group pension plan consists of 5 people ($m_t = 5$), the normal cost that will be paid by each group member is:

$$FIL(NC)_{t_j} = \frac{1}{5} \left( B_{65_1} \ddot{a}_{65} \frac{D_{65_1}}{N_{26_1} - N_{65_1}} + B_{65_2} \ddot{a}_{65} \frac{D_{65_2}}{N_{27_2} - N_{65_2}} + B_{65_3} \ddot{a}_{65} \frac{D_{65_3}}{N_{28_3} - N_{65_3}} + B_{65_4} \ddot{a}_{65} \frac{D_{65_4}}{N_{30_4} - N_{65_4}} + B_{65_5} \ddot{a}_{65} \frac{D_{65_5}}{N_{31_5} - N_{65_5}} \right).$$

$$FIL(NC)_{t_j} = Rp3.646.067, -.$$

So, the amount of normal cost that must be paid by each participant is the same, which is equal to Rp3.646.067, – per year or by Rp303.839, – per month.

4. When $t = 34$
When the service period reaches 34 years, the group pension plan consists of 4 people ($m_t = 4$), the normal cost that will be paid by each group member is:

$$FIL(NC)_{t_j} = \frac{1}{4} \left( B_{65_1} \ddot{a}_{65} \frac{D_{65_1}}{N_{26_1} - N_{65_1}} + B_{65_2} \ddot{a}_{65} \frac{D_{65_2}}{N_{27_2} - N_{65_2}} + B_{65_3} \ddot{a}_{65} \frac{D_{65_3}}{N_{28_3} - N_{65_3}} + B_{65_4} \ddot{a}_{65} \frac{D_{65_4}}{N_{30_4} - N_{65_4}} \right).$$

$$FIL(NC)_{t_j} = Rp3.474.408, -.$$

So, the amount of normal cost that must be paid by each participant is the same, which is equal to Rp3.474.408, – per year or by Rp289.534, – per month.

5. When $t = 35$ to $t = 36$
At 35-36 years, the group pension plan consists of 3 people ($m_t = 3$), then the normal cost that will be paid by each group member is:

$$FIL(NC)_{t_j} = \frac{1}{3} \left( B_{65_1} \ddot{a}_{65} \frac{D_{65_1}}{N_{26_1} - N_{65_1}} + B_{65_2} \ddot{a}_{65} \frac{D_{65_2}}{N_{27_2} - N_{65_2}} + B_{65_3} \ddot{a}_{65} \frac{D_{65_3}}{N_{28_3} - N_{65_3}} \right).$$

$$FIL(NC)_{t_j} = Rp3.286.645, -.$$

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So, the amount of normal cost that must be paid by each participant is the same, which is equal to \( Rp3.286.645 \), – per year or by \( Rp273.887 \), – per month.

6. When \( t = 37 \)

When the service period reaches 37 years, the group pension plan consists of 2 people (\( m_t = 2 \)), the normal cost that will be paid by each group member is:

\[
FIL(NC)_{t_j} = \frac{1}{2} \left( B_{65_1} \bar{a}_{65} \frac{D_{65_1}}{N_{26_1} - N_{65_1}} + B_{65_2} \bar{a}_{65} \frac{D_{65_2}}{N_{27_2} - N_{65_2}} \right)
\]

So, the amount of normal cost that must be paid by each participant is the same, which is equal to \( Rp3.173.102 \), – per year or by \( Rp264.425 \), – per month.

7. When \( t = 38 \)

When the service period reaches 38 years, the pension plan only consists of 1 member, the normal cost that will be paid is:

\[
FIL(NC)_{t_j} = \frac{1}{2} (6346204,65) = Rp3.173.102, –.
\]

It is obtained that amount of contributions that must be paid by the participant is \( Rp3.065.294 \), – per year or by \( Rp255.441 \), – per month. At a period of more than 38 years, the group member, namely \( j = 1 \) has entered his retirement period and stopped paying normal cost. This means that the group pension program has 0 people left, so the normal cost payment stops.

**IV. CONCLUSIONS**

Normal cost using the Frozen Initial Liability (FIL) method of each participant is equal for each month, and will decrease when there are members who have reached their retirement age. At the time of \( t = 0 \) to \( t = 30 \), the normal cost is constant because at that time all members of the pension fund program are still active. When the working period enters 31 years, one by one the participants begin to retire so that the normal cost begins to decrease. The greater the number of members in a pension fund program group, the greater the normal cost value that must be paid every month.

**V. REFERENCES**


