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PRIME LABELING OF SOME WEB GRAPHS WITHOUT CENTER

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Abstract. The prime labeling of a graph G of order n is a bijection function from the set of vertices in G to the set of the first n positive integers, such that any two adjacent points in G have labels that are coprime to each other. In this paper we discuss the primality of the graph $W_0(2, n)$ along with its combinations with similar graphs and various types of edges subdivisions in the graph $W_0(2, n)$. Moreover, it is also presented the necessary and sufficient conditions for the graph to be prime.
Keywords: Prime Labeling, Web Graph without Center, Independence Number.

I. INTRODUCTION

In mathematics, graph theory is a branch of science that studies graphs and their properties. A graph [11] is defined as a pair $(V(G), E(G))$ where $V(G)$ is a non-empty finite set whose elements are called vertices, and $E(G)$ is a finite set of unordered pairs of distinct elements from $V(G)$ called edges.

One of the topics in graph theory is graph labeling. Based on a survey by Gallian [3], graph labeling was first introduced in the mid-1960s, and in the intervening years over 200 graph labelings techniques have been studied in over 3000 papers. Graph labeling [10] is a mapping that assigns elements of a graph to numbers (usually positive or non-negative numbers). Typically, the elements being mapped are either points or edges. If the elements being mapped are points, the labeling is called vertex labeling.

One type of vertex labeling is prime labeling. The idea of prime labeling originated from Entriger's conjecture in 1980, which stated that every tree graph can have a prime labeling. The development of this conjecture was last discovered in 2011 by Haxell et al. [4], who proved that all tree graphs with a sufficiently large number of vertices are prime graphs. According to this conjecture, Tout et al. [9] introduced prime labeling in a paper published in 1982. Recall that a vertex labeling $f : V(G) \to \{1, 2, \ldots, n\}$, where f is a bijection function is said to be **prime labeling** of a graph G if for every $x, y \in V(G)$ with $xy \in E(G)$, $f(x)$ and $f(y)$ are relatively prime, in other words, $gcd(f(x), f(y)) = 1$. A graph G that admits a prime labeling is called a prime graph on G.

Recall that the set $I \subseteq V(G)$ is called as independent set in G if for any $x, y \in I$, $xy \notin E(G)$. The size of largest independent set in G is called the independence number of G and is denoted $\beta_0(G)$. In [6], it is stated that in order a graph G to be prime, necessarily $\beta_0(G) \geq \left\lfloor \dfrac{|V(G)|}{2} \right\rfloor$.

If $n \geq 4$ then the complete graph K_n is not prime [7]. On the other hand, the cycle C_n is a prime graph [1], which may be observed by labeling the sequence of its adjacent vertices by the sequence of consecutive integers. The disjoint union graph $C_m \cup C_n$ of C_m and C_n is a prime graph if and only if mn is even. This fact motivated Kansagara in [5] and [6] for similar

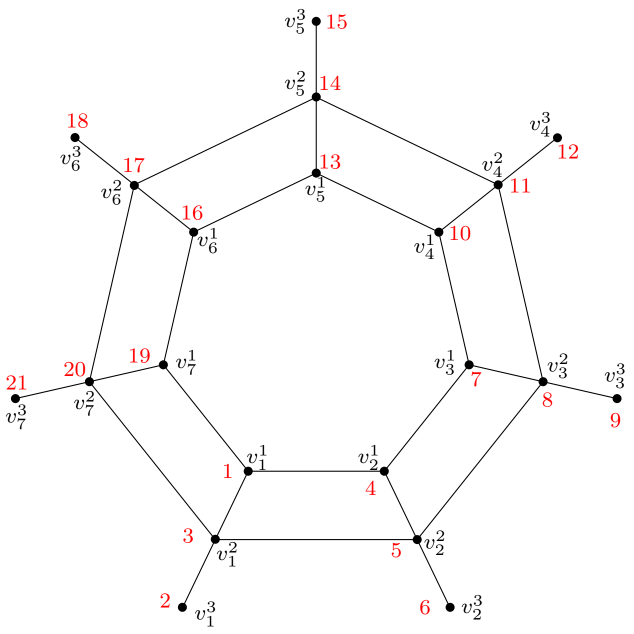

Figure 1. Prime labeling of graph $W_0(2, 7)$

results for the web graph without center. Inspired by the work of Kansagara [5], we then modify the web graphs without center $W_0(2, n)$ by performing a subdivision on some specific edges in these graphs. Furthermore, we study the primality of these graphs and the disjoint union of two of them as well. The primality of the disjoint union of the graph $W_0(2, n)$ with the Jahangir graph $J_{m,k}$ previously given in [6] is also examined. We found a counter example of the result given in [6]. Furthermore, a more accurate version of the theorem is presented.

II. DISCUSSION

Throughout this paper, for any positive number $n \geq 3$, the web graph without center $W_0(2, n)$ is a graph with vertex and edge sets as follows

$$\begin{aligned} V(W_0(2,n)) &= \{v_j^t \mid t = 1,2,3\,; j = 1,2,\ldots,n\} \\ E(W_0(2,n)) &= \{v_j^t v_j^{t+1} \mid t = 1,2\,; j = 1,2,\ldots,n\} \cup \\ &\qquad\qquad \{v_1^t v_n^t, v_j^t v_{j+1}^t \mid t = 1,2\,; j = 1,2,\ldots,n-1\}. \end{aligned}$$

Observe that for $t = 1$, v_j^t's represent the vertices of the inner cycle, for $t = 2$, v_j^t's represent the vertices of the outer cycle, and for $t = 3$, v_j^t's represent the pendent vertices. Moreover $|V(W_0(2, n))| = 3n$ and $|E(W_0(2, n))| = 4n$. Based on the order and the size of $W_0(2, n)$, the following Lemma 2.1 has been proven in [6].

Lemma 2.1 [6] *Let $n \geq 3$ be any positive integer. It holds that*

$$\beta_0(W_0(2,n)) = \left\lfloor \frac{3n}{2} \right\rfloor.$$

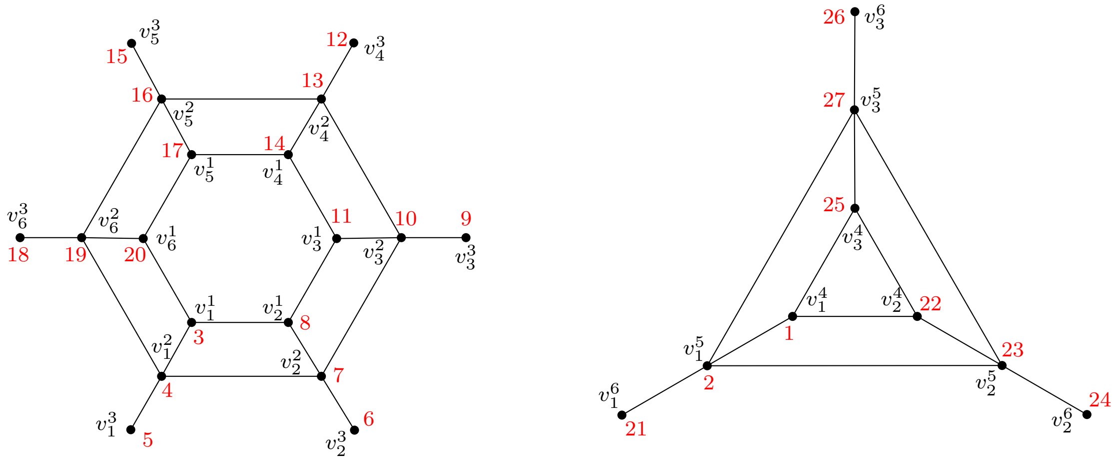

Figure 2. Prime labeling of graph $W_0(2,6) \cup W_0(2,3)$

Recall that for any positive positive numbers a and b, it follows that gcd $(a, b) =$ gcd (b, a) and gcd $(a, b) =$ gcd $(a \pm kb, b)$ for any positive integer k and $a - kb > 0$.

The following theorem gives necessary and sufficient condition for the union of two web graphs without center to be prime.

Theorem 2.1 [6] *For any positive integers $m, n \geq 3$, the graph $W_0(2, m) \cup W_0(2, n)$ is prime if and only if mn is even.*

The example of prime labeling of graph $W_0(2, m) \cup W_0(2, n)$ is shown in Figure 2.

Now, for any given graph G, and a specific edge subset A of $E(G)$ we define a graph G^A as follows.

Definition 2.1 *Given $a, b \in V(G)$ such that $ab \in E(G)$. The subdivision of the edge ab in G is an operation on the graph G that adds a new vertex c on the edge ab resulting in the removal of the edge ab and the formation of two new edges, named ac and cb. If the subdivision operation is applied to all edges in $A \subseteq E(G)$, the resulting graph is denoted as G^A.*

Particularly for the web graph without center $W_0(2, n)$ and a particular edge subset C of $W_0(2, n)$, the primality of $W_0^C(2, n)$ is given in the subsequent theorem.

Theorem 2.2 *Let C be the set of all edges joining the inner and outer cycles in the graph $W_0(2, n)$. The graph $W_0^C(2, n)$ is a prime graph for any positive integer $n \geq 3$.*

Proof. Given $W_0^C(2, n)$ by

$$
\begin{aligned}
V(W_0^C(2, n)) &= \{v_j^t \mid t = 1, 2, 3, 4; j = 1, 2, \ldots, n\} \\
E(W_0^C(2, n)) &= \{v_j^t v_j^{t+1} \mid t = 1, 2, 3; j = 1, 2, \ldots, n\} \cup \\
&\quad \{v_1^t v_n^t, v_j^t v_{j+1}^t \mid t = 1, 3; j = 1, 2, \ldots, n - 1\}
\end{aligned}
$$

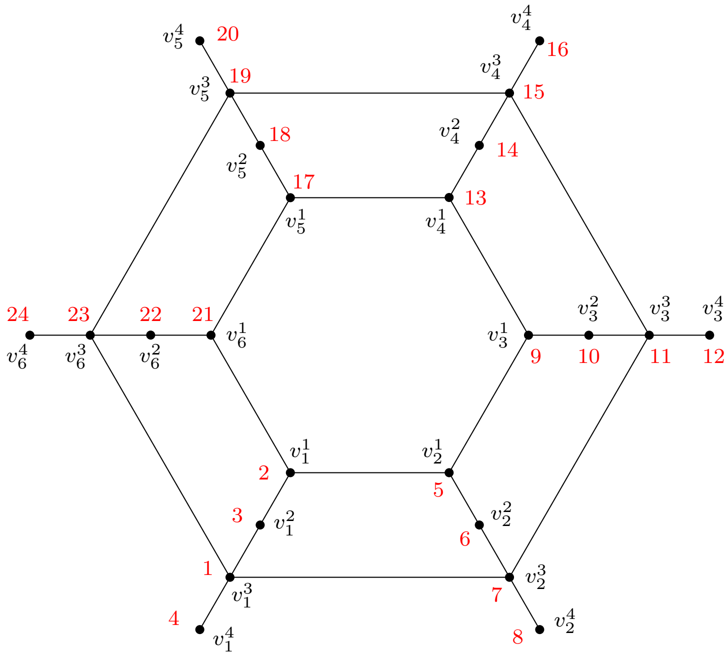

Figure 3. Prime labeling of graph $W_0^C(2,6)$

Let $G = W_0^C(2,n)$. Thus $|V(G)| = 4n$ and $|E(G)| = 5n$. Define $f : V(G) \to \{1,2,\ldots,4n\}$, Therefore, for every $i = 2,3,\ldots,n$,

$$\begin{array}{llll} f(v_1^1) = 2 & f(v_1^2) = 3 & f(v_1^3) = 1 & f(v_1^4) = 4 \\ f(v_i^1) = 4i-3 & f(v_i^2) = 4i-2 & f(v_i^3) = 4i-1 & f(v_i^4) = 4i. \end{array}$$

Hence, gcd $(f(v_1^1), f(v_1^2))$ = gcd $(2,3) = 1$, gcd $(f(v_1^2), f(v_1^3))$ = gcd $(3,1) = 1$, gcd $(f(v_1^3), f(v_1^4))$ = gcd $(1,4) = 1$, gcd $(f(v_1^1), f(v_n^1))$ = gcd $(2, 4n-3) = 1$, gcd $(f(v_1^3), f(v_n^3))$ = gcd $(1, 4n-1) = 1$, gcd $(f(v_1^1), f(v_2^1))$ = gcd $(2,5) = 1$, gcd $(f(v_1^3), f(v_2^3))$ = gcd $(1,7) = 1$. For $i = 2,3,\ldots,n-1$, gcd $(f(v_i^1), f(v_{i+1}^1))$ = gcd $(4i-3, 4i+1) = 1$ and gcd $(f(v_i^3), f(v_{i+1}^3))$ = gcd $(4i-1, 4i+3) = 1$. Rest of the adjacent vertices are labeled with consecutive integers. Hence, $f$is a prime labeling on $W_0^C(2,n)$, so $W_0^C(2,n)$ is a prime graph. □

The example of prime labeling of graph $W_0^C(2,n)$ is shown in Figure 3.

In the following theorem, we prove that the the union of two web graphs $W_0^C(2,m)$ and $W_0^C(2,n)$ is a prime graph.

Theorem 2.3 *For any positive integers $m,n \geq 3$, the graph $W_0^C(2,m) \cup W_0^C(2,n)$ is a prime.*

Proof. Given $W_0^C(2,m)$ with

$$V(W_0^C(2,m)) = \{v_j^t \mid t = 1,2,3,4\,;\, j = 1,2,\ldots,m\}$$
$$E(W_0^C(2,m)) = \{v_j^t v_j^{t+1} \mid t = 1,2,3\,;\, j = 1,2,\ldots,m\} \cup \{v_1^t v_m^t, v_j^t v_{j+1}^t \mid t = 1,3\,;\, j = 1,2,\ldots,m-1\}$$

and $W_0^C(2,n)$ with

$$\begin{aligned} V(W_0^C(2,n)) &= \{v_j^t \mid t=5,6,7,8\,;j=1,2,\dots,n\} \\ E(W_0^C(2,n)) &= \{v_j^t v_j^{t+1} \mid t=5,6,7\,;j=1,2,\dots,n\} \cup \\ &\quad \{v_1^t v_n^t, v_j^t v_{j+1}^t \mid t=5,7\,;j=1,2,\dots,n-1\}. \end{aligned}$$

Let $G = W_0^C(2,m) \cup W_0^C(2,n)$. Thus $|V(G)| = 4m+4n$ and $|E(G)| = 5m+5n$. Let us define $f : V(G) \to \{1,2,\dots,4m+4n\}$ with for every $i = 3,4,\dots,m$ and $j = 2,3,\dots,n$,

$$\begin{array}{llll} f(v_1^1) = 8 & f(v_1^2) = 5 & f(v_1^3) = 4 & f(v_1^4) = 3 \\ f(v_2^1) = 9 & f(v_2^2) = 10 & f(v_2^3) = 7 & f(v_2^4) = 6 \\ f(v_i^1) = 4i-1 & f(v_i^2) = 4i & f(v_i^3) = 4i+1 & f(v_i^4) = 4i+2 \\ f(v_1^5) = 2 & f(v_1^6) = 4m+3 & f(v_1^7) = 1 & f(v_1^8) = 4m+4 \\ f(v_j^5) = 4m+4j-3 & f(v_j^6) = 4m+4j-2 & f(v_j^7) = 4m+4j-1 & f(v_j^8) = 4m+4j. \end{array}$$

On graph $W_0^C(2,m)$, gcd $(f(v_1^1), f(v_1^2))$ = gcd $(8,5) = 1$, gcd $(f(v_1^2), f(v_1^3))$ = gcd $(5,4) = 1$, gcd $(f(v_1^3), f(v_1^4))$ = gcd $(4,3) = 1$, gcd $(f(v_1^1), f(v_m^1))$ = gcd $(8, 4m-1) = 1$, gcd $(f(v_1^3), f(v_m^3))$ = gcd $(4, 4m-3) = 1$, gcd $(f(v_1^1), f(v_2^1))$ = gcd $(8,9) = 1$, gcd $(f(v_1^3), f(v_2^3))$ = gcd $(4,7) = 1$, gcd $(f(v_2^1), f(v_3^1))$ = gcd $(9,11) = 1$, and gcd $(f(v_2^3), f(v_3^3))$ = gcd $(7,13) = 1$. For $i = 3,4,\dots,m$, gcd $(f(v_i^1), f(v_{i+1}^1))$ = gcd $(4i-1, 4i+3) = 1$ and gcd $(f(v_i^3), f(v_{i+1}^3))$ = gcd $(4i+1, 4i+5) = 1$.

On graph $W_0^C(2,n)$, gcd $(f(v_1^5), f(v_1^6))$ = gcd $(2, 4m+3) = 1$, gcd $(f(v_1^6), f(v_1^7))$ = gcd $(4m+3, 1) = 1$, gcd $(f(v_1^7), f(v_1^8))$ = gcd $(1, 4m+4) = 1$, gcd $(f(v_1^5), f(v_n^5))$ = gcd $(2, 4m+4n-3) = 1$, gcd $(f(v_1^7), f(v_n^7))$ = gcd $(1, 4m+4n-1) = 1$, gcd $(f(v_1^5), f(v_2^5))$ = gcd $(2, 4m+5) = 1$, and gcd $(f(v_1^7), f(v_2^7))$ = gcd $(1, 4m+7) = 1$. For $j = 2,3,\dots,n$, gcd $(f(v_j^5), f(v_{j+1}^5))$ = gcd $(4m+4j-3, 4m+4j+1) = 1$ and gcd $(f(v_j^3), f(v_{j+1}^3))$ = gcd $(4m+4j-1, 4m+4j+3) = 1$.

Rest of the adjacent vertices are labeled with consecutive integers. Hence, f is a prime labeling on G. □

The example of prime labeling of graph $W_0^C(2,m) \cup W_0^C(2,n)$ is shown in Figure 4.

Theorem 2.4 *Let D be the set of all edges of the inner cycles and outer cycles in the graph $W_0(2,n)$. The graph $W_0^D(2,n)$ is a prime for all $n \geq 3$.*

Proof. Given graph $W_0^D(2,n)$ as

$$\begin{aligned} V(W_0^D(2,n)) &= \{v_j^t \mid t=1,2,3; j=1,2,\dots,n\} \cup \\ &\quad \{x_j^t \mid t=1,2; j=1,2,\dots,n\} \\ E(W_0^D(2,n)) &= \{v_j^t v_j^{t+1}, v_j^t x_j^t \mid t=1,2; j=1,2,\dots,n\} \cup \\ &\quad \{v_n^t x_1^t, v_j^t x_{j+1}^t \mid t=1,2; j=1,2,\dots,n-1\} \end{aligned}$$

Let $G = W_0^D(2,n)$. Thus, $|V(G)| = 5n$ and $|E(G)| = 6n$. Define $f : V(G) \to \{1,2,\dots,5n\}$,

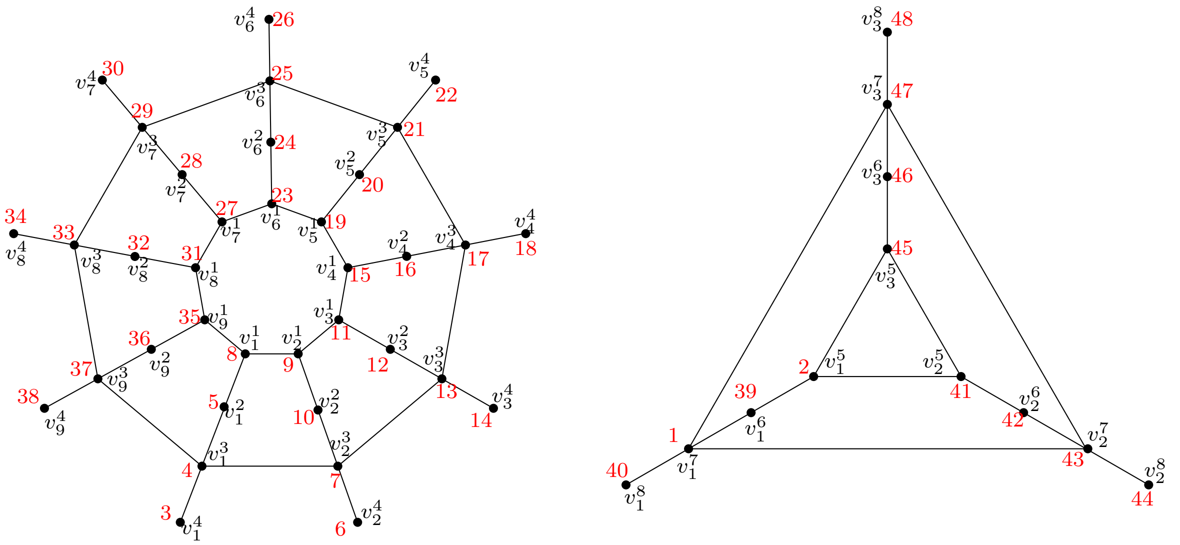

Figure 4. Prime labeling of graph $W_0^C(2,9) \cup W_0^C(2,3)$

so for every $i = 1, 3, \ldots, 2\left\lfloor \frac{n+1}{2} \right\rfloor - 1$ and $j = 2, 4, \ldots, 2\left\lfloor \frac{n}{2} \right\rfloor$,

$$
\begin{aligned}
f(v_i^1) &= 5i - 4 & \qquad f(v_j^1) &= 5j - 3\\
f(v_i^2) &= 5i - 2 & f(v_j^2) &= 5j - 1\\
f(v_i^3) &= 5i & f(v_j^3) &= 5j\\
f(x_i^1) &= 5i - 3 & f(x_j^1) &= 5j - 4\\
f(x_i^2) &= 5i - 1 & f(x_j^2) &= 5j - 2.
\end{aligned}
$$

For $i = 1, 3, \ldots, 2\left\lfloor \frac{n+1}{2} \right\rfloor - 1$, then gcd $(f(v_i^1), f(v_i^2)) =$ gcd $(5i-4, 5i-2) = 1$, gcd $(f(v_i^2), f(v_i^3)) =$ gcd $(5i-2, 5i) = 1$, gcd $(f(v_i^1), f(x_i^1)) =$ gcd $(5i-4, 5i-3) = 1$, and gcd $(f(v_i^2), f(x_i^2)) =$ gcd $(5i-2, 5i-1) = 1$.

For $i = 1, 3, \ldots, 2\left\lfloor \frac{n}{2} \right\rfloor - 1$, it is clear that $i+1$ is even, so gcd $(f(v_i^1), f(x_{i+1}^1)) =$ gcd $(5i-4, 5(i+1)-4) = 1$, and gcd $(f(v_i^2), f(x_{i+1}^2)) =$ gcd $(5i-2, 5(i+1)-2) = 1$. For n is odd, gcd $(f(v_n^1), f(x_1^1)) =$ gcd $(5n-4, 2) = 1$ and gcd $(f(v_n^2), f(x_1^2)) =$ gcd $(5n-2, 4) = 1$.

For $j = 2, 4, \ldots, 2\left\lfloor \frac{n}{2} \right\rfloor$, then gcd $(f(v_j^1), f(v_j^2)) =$ gcd $(5j-3, 5j-1) = 1$, gcd $(f(v_j^2), f(v_j^3)) =$ gcd $(5j-1, 5j) = 1$, gcd $(f(v_j^1), f(x_j^1)) =$ gcd $(5j-3, 5j-4) = 1$, and gcd $(f(v_j^2), f(x_j^2)) =$ gcd $(5j-1, 5j-2) = 1$.

For $j = 2, 4, \ldots, 2\left\lfloor \frac{n+1}{2} \right\rfloor - 2$, it is clear that $j+1$ is odd, so gcd $(f(v_j^1), f(x_{j+1}^1)) =$ gcd $(5j-3, 5(j+1)-3) = 1$, and gcd $(f(v_j^2), f(x_{j+1}^2)) =$ gcd $(5j-1, 5(j+1)-1) = 1$. For n is even, gcd $(f(v_n^1), f(x_1^1)) =$ gcd $(5n-3, 2) = 1$ and gcd $(f(v_n^2), f(x_1^2)) =$ gcd $(5n-1, 4) = 1$. $\square$

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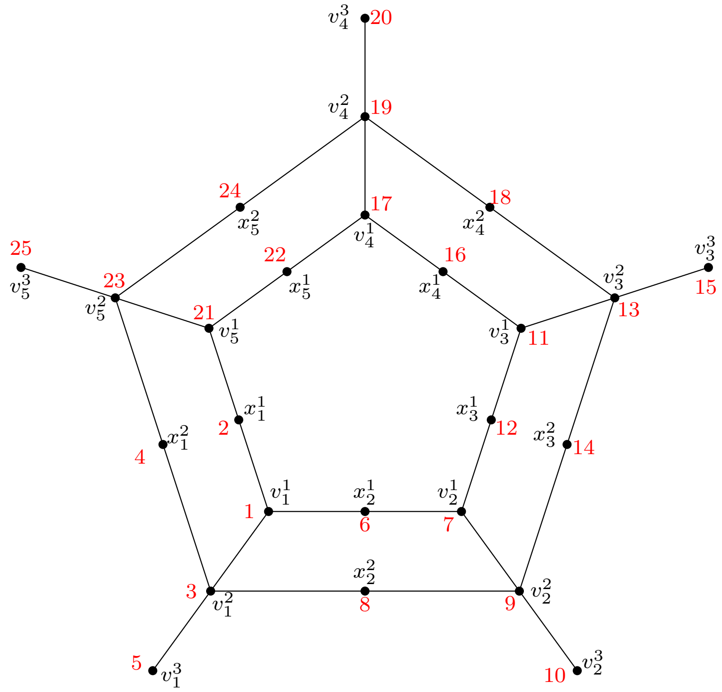

Figure 5. Prime labeling of graph $W_0^D(2,5)$

The example of prime labeling of graph $W_0^D(2,n)$ is shown in Figure 5.

Definition 2.2 *The Jahangir graph $J_{m,k}$ is a graph with*

$$
\begin{aligned}
V(J_{m,k}) &= \{x_j \mid j = 0, 1, 2, \ldots, mk\} \\
E(J_{m,k}) &= \{x_1 x_m, x_j x_{j+1} \mid j = 1, 2, \ldots, mk-1\} \cup \\
&\quad\ \{x_0 x_{jk} \mid j = 1, 2, \ldots, m\}.
\end{aligned}
$$

In Theorem 2.5 given in [6], necessary and sufficient conditions for the labeling of union of $W_0(2,n)$ and Jahangir graph $J_{m,k}$ with a prime labeling is given as follows.

Theorem 2.5 [Kansagara (2021)] *The graph $W_0(2,n) \cup J_{m,k}$ is prime if and only if one of the following and **only** if one of the following two conditions hold.*

1. *n and m both are even.*
2. *k is even.*

The necessary and sufficient conditions for the union of graph $W_0(2,n)$ and $J_{m,k}$ in Theorem 2.5 is not entirely correct. Figure 6. is one of the counter example to the theorem. We find that with $n = 6, m = 2$, and $k = 4$, the graph $W_0(2,n) \cup J_{m,k}$ is prime. The more accurate theorem is given in Theorem 2.6 as follows.

Theorem 2.6 *The graph $W_0(2,n) \cup J_{m,k}$ is prime if and only if either one or both of the following statements hold.*

1. *n and m both are even.*

2. *k is even.*

Proof. Let $G = W_0(2,n) \cup J_{m,k}$. Thus $|V(G)| = 3n + mk + 1$. We consider several cases.

1. Statement 1 and statement 2 hold.
   Define $f : V(G) \to \{1, 2, \ldots, 3n + mk + 1\}$, such that for every $i = 2, 3, \ldots, n$ and $j = 2, 3, \ldots, mk$,

$$\begin{array}{lll} f(v_1^1) = 3 & f(v_1^2) = 4 & f(v_1^3) = 5 \\ f(v_i^1) = 3i + 2 & f(v_i^2) = 3i + 1 & f(v_i^3) = 3i \\ f(x_0) = 1 & f(x_1) = 2 & f(x_j) = 3n + j + 1. \end{array}$$

   Based on the proof of Theorem 2.1, it is clear that vertices in $W_0(2,n)$ have a prime labeling. Furthermore, for every $i = 1, 2, \ldots, m$, gcd $(f(x_0), f(x_{ik}))$ = gcd $(1, 3n + ik + 1) = 1$, gcd $(f(x_1), f(x_2))$ = gcd $(2, 3n + 3) = 1$, and gcd $(f(x_1), f(x_{mk}))$ = gcd $(2, 3n + mk + 1) = 1$. The remaining adjacent vertices are labeled with consecutive integers. Hence, f is a prime labeling on G so that graph G is a prime graph.

2. Statement 1 holds and statement 2 does not hold.
   We can choose f as in case 1. Since there are no changes in the labeling pattern for the formed edges, f is a prime labeling. Therefore, for k is odd and m, n is even, G is a prime graph.

3. Statement 2 holds and statement 1 does not hold.
   It can be easily shown that for even n, f as in case 1 is a prime labeling. Hence the graph G is prime. For odd n, f is no longer a prime labeling as $3n+3$, $3n+mk+1$, and $3n+1$ are all even, so that

$$\begin{aligned} &\text{gcd } (f(x_1), f(x_{mk})) = \text{gcd } (2, 3n + mk + 1) = 2 \\ &\text{gcd } (f(x_1), f(x_2)) = \text{gcd } (2, 3n + 3) = 2 \\ &\text{gcd } (f(v_1^2), f(v_n^2)) = \text{gcd } (4, 3n + 1) = 2 \text{ or } 4. \end{aligned}$$

   However, we can modify f into a new labeling, namely $g : V(G) \to \{1, 2, \ldots, 3n + mk + 1\}$ defined by

$$g(v) = \begin{cases} f(v) & \text{for } v \notin \{v_n^2, v_n^3\} \\ f(v_n^2) & \text{for } v = v_n^3 \\ f(v_n^3) & \text{for } v = v_n^2 \end{cases}$$

   for every $v \in V(W_0(2,n))$, for every $i = 1, 2, \ldots, k-1$ and $j = 1, 2, \ldots, (m-1)k$,

$$\begin{array}{ll} g(x_0) = 2 & g(x_i) = 3n + (m-1)k + 2 + i \\ g(x_k) = 1 & g(x_{k+j}) = 3n + 2 + j. \end{array}$$

   It is easily check that g is prime labeling, so that graph G is a prime.

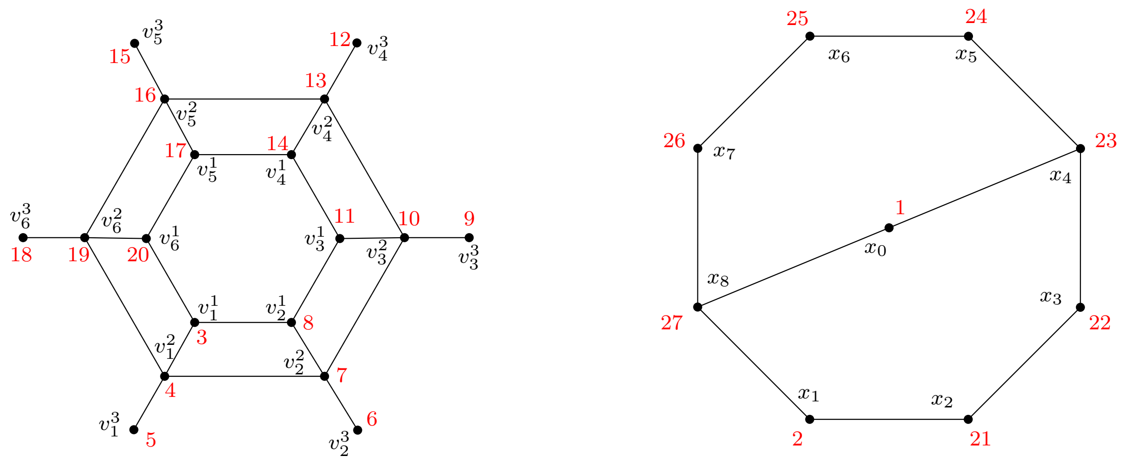

Figure 6. Prime labeling of graph $W_0(2,6) \cup J_{2,4}$

4. Both statement does not holds.
   Clearly k is odd and the values of n and m are either one or both is odd. Thus,

$$\beta_0(G) \leq \begin{cases} \frac{3n-1}{2} + \frac{mk}{2} & \text{if } n \text{ is odd and } m \text{ is even} \\ \frac{3n}{2} + \frac{mk-1}{2} & \text{if } n \text{ is even and } m \text{ is odd} \\ \frac{3n-1}{2} + \frac{mk-1}{2} & \text{if } n \text{ and } m \text{ both are odd.} \end{cases}$$

   Therefore, in every condition we have $\beta_0(G) < \left\lfloor \frac{3n+mk+1}{2} \right\rfloor = \left\lfloor \frac{|V(G)|}{2} \right\rfloor$. As a result, G is not a prime graph.

$\square$

III. CONCLUSIONS

Based on the discussion above, it can be concluded that any modified web graphs without center $W_0^C(2,n)$, $W_0^D(2,n)$ are prime, and the disjoint union of any two modified web graphs without center $W_0^C(2,n) \cup W_0^C(2,m)$ is also prime for all $n \geq 3$.

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