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BONUS MALUS SYSTEM FOR MOTORIZED VEHICLE INSURANCE USING GEOMETRIC DISTRIBUTIONS AND WEIBULL DISTRIBUTIONS

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Abstract . The bonus malus system is one of the systems used to determine the premium amount for the next period based on the claim history of the policyholder. If the policyholder has no claims history or did not file a claim in the previous year, then the policyholder will get a bonus or in other words will get a reduction in the premium rate in the following period. Meanwhile, if the policyholder has a history of claims in the previous year, then the policyholder will be subject to a malus or must pay an increase in the premium rate in the following period. The purpose of this study is to calculate motor vehicle insurance premiums using the classic and optimal bonus malus method which takes into account the frequency of claims with a geometric distribution and the size of claims with a Weibull distribution. The results of this study indicate that the optimal bonus malus system is fairer for policyholders who renew their policies because the premium paid by the policyholder depends on the number of claims and the size of the claim, so that each policyholder will pay a different premium according to the number of claims.
Keywords : Premium, Malus Bonus System, Claim Frequency Distribution, Claim Size Distribution.

I. INTRODUCTION

With relatively rapid technological advances including motorized vehicles which are always increasing every year and the possibility of risk will increase. An increase in the number of motorized vehicles raises the possibility of a hazard or risk. For overcome this risk, owner vehicle needs protection for the vehicles. This situation provides an opportunity for the company insurance for offer product protecting the owner the vehicle in the form of repairs in case of damage or replacement if the vehicle lost. Insurance according to article 246 of the Commercial Law Code (KUHD) is a contract between two or more parties, in which the insurance company protects the policyholder from damage caused by unexpected events by receiving a premium to compensate the policyholder's losses. [1]. Contracts are usually calculated according to the insurance system used by the insurance company. Every company insurance applied system payment premium in a manner different. One of the systems u sed in vehicle motorized insurance is the *bonus system malus*.

Bonus-malus is system which give penalty if happen at least filed one claim and premium reduction if you do not file a claim in the previous year [2]. In other words, the *bonus* is a reduction in premium rate payments if the insured does not make a claim in the previous year.

Whereas *malus* is enhancement payment rates premium Because the insured filed claim on year before [3]. In general, there are two types of *bonus systems* , namely *the bonus system* that is onlybased on the number of claims and a system based on the number of claims and the amount of claims [3]. According to [4] a bonus system *malus* that pays premiums only involving the number of claims filed by the insured is called the classic *bonus malus system* . Meanwhile, the *bonus malus* system , where premium payments are based on the number and size of claims submitted by the insured in the previous year is called the optimal *bonus malus* system .

Statistically, premium rates can be calculated by modeling data regarding the number of claims and the amounts of claims. The number of claims is usually modeled by standard discrete distributions such as the Poisson, geometric and negative binomial distributions [5]. The commonly used claim modeling is the Poisson distribution. However, in practice, claims data often shows overdispersion [6]. As an alternative in overcoming overdispersion of data, you can use a mixed Poisson distribution [7]. One of the mixed Poisson distributions commonly used in frequency claims is the geometric distribution which is a mixed Poisson-exponential distribution. [8]. As for the magnitude of claims, it is usually modeled with continuous distributions such as the lognormal, gamma, exponential, Weibull and Pareto distributions [5]. The distribution that is commonly used to solve various problems related to the insurance claim data model is the Weibull distribution. The Weibull distribution is often used because it describes the whole data clearly, especially test data and data modeling [9].

Several studies related to the bonus malus system have been carried out in the automotive industry, in research conducted by Si, J, He (2021), applying learning methods ensemble for prediction of claims [18]. The development of the bonus malus method has also been carried out with fuzzy, computational and applied approaches [13-17]. Therefore, on this study will use the geometric distribution and the Weibull distribution as distributions frequency of claims and distribution of claims in determining premium rates using the system
bonus malus on motor vehicle insurance.

II. METHODOLODY

The data used as a case study in this study is in the form of secondary data obtained from existing sources with the aim of the research, namely the calculation of insurance premiums that must be paid by the insured to the insurance company using the bonus malus system method. The data obtained is in the form of the number of claims and the amount of claims submitted by each policyholder. The steps of the analytical method in this study are:

1. Determining the premium formula based on the frequency of claims with the classical *bonus malus system* by:
   a. Poison function: the frequency of claims submitted is expressed by n, which is assumed to have a Poisson (P) distribution with parameters λ
   b. Exponential Function: by λ stating the risk size of each policyholder and λ exponential distribution with parametersθ
   c. Determine the unconditional distribution of claim frequencies by integrating the two equations $p(k) = \int_0^\infty p(k;\lambda)u(\lambda)d\lambda$that form the probability mass function for the geometric distribution with parameter θ.
   d. Estimating the parameters of the classical *bonus malus system* , namely $\lambda$by Bayes theorem and the prior structure function of a collection of claims. So that the posterior distribution of the claim frequency parameters is obtained:

i. *Prior* distribution

The joint probability density function of the total claim frequency is:

$$p(k_1, k_2, \dots, k_t|\lambda) = \frac{e^{-\lambda t}\lambda^K}{\prod_{i=1}^{t} k_t!}$$

and the probability λn density function of is:

$$u(\lambda) = \theta e^{-\lambda\theta}$$

ii. *Posterior* spread

$$U(|\lambda|k_1, k_2, \dots, k_t) \propto p(k_1, k_2, \dots, k_t|\lambda)u(\lambda)$$
$$\propto e^{-\lambda t}\lambda^K e^{-\theta\lambda}$$
$$\propto e^{-\lambda(\theta+t)}\lambda^K$$

e. Determine the expected value of the frequency of claims that occur for each insured.
f. Determine the premium to be paid by each insured in the year $t + 1$.

2. Determining the model of premium formula based on the frequency of claims and the ammount of claims with the optimal *bonus malus system* by:
   a. Exponential function: the amount of the proposed claim can be expressed with $c$an exponential distribution with parameters λ. So that the unconditional distribution of the amount of claims for each policyholder is the Weibull distribution. The Weibull distribution of the distribution $c$is:
   $$f(x|\theta) = \theta e^{-\theta x}$$
   b. Levy function: by λstating the average magnitude, the parameter $\theta$is Levy distribution with parameter c, with the probability density function as follows :
   $$u(\theta) = \frac{c}{2\sqrt{\pi\theta^3}}\exp\left(-\frac{c^2}{4\theta}\right) \quad ; c > 0, \theta > 0.$$
   c. Determine the unconditional distribution of claim magnitudes by integrating the two equations $f(x) = \int_0^\infty f(x|\theta)u(\theta)d\theta$that form the probability density function for the Weibull distribution.
   d. Estimating the parameters of the optimal *malus bonus system* is the Bayesian approach with a quadratic loss function. So that the posterior distribution of the claim size parameters will be obtained, namely:
   $$U(\theta|x_1, x_2, \dots x_K) = \frac{f(x_1, x_2, \dots, x_n|\theta)u(\theta)}{\int f(x_1, x_2, \dots, x_n|\theta)u(\theta)d\theta}$$
   $$= \frac{\theta^{K-\frac{3}{2}}e^{-\left(\frac{c^2}{4\theta}+\theta\left(\sum_{i=1}^{K} x_i\right)\right)}}{\int_0^\infty \theta^{K-\frac{3}{2}}e^{-\left(\frac{c^2}{4\theta}+\theta\left(\sum_{i=1}^{K} x_i\right)\right)}d\theta}$$
   e. Determine the expected value of the posterior distribution of the amount of claims that occur for each insured.
   f. Determine the premium to be paid by each insured in the year$t + 1$.

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g. Determine the premium to be paid by the insured when not making a claim$(K = 0)$.

3. Implementing the premium formula on the classic *malus bonus system* and the optimal *malus bonus system* .

III. RESULTS AND DISCUSSION

3.1 Classic *Malus Bonus* System premium modeling

According to [12] the *bonus malus* system claims frequency distribution is modeled by the geometric distribution which is a mixed distribution of the Poisson distribution with parameters $\lambda$and $k$is the frequency of claims, then the probability density function is:

$$p(k;\lambda) = \frac{e^{-\lambda}\lambda^k}{k!}\ , untuk\ k = 0,1,2,\ldots n\ dan\ \lambda > 0. \tag{3.1}$$

Where has λ an exponential distribution with parameters θ, so that the probability density function is:

$$u(\lambda) = \theta e^{-\lambda\theta}\ , dengan\ \lambda > 0\ dan\ \theta > 0. \tag{3.2}$$

assumed that k_i shows the frequency of claims that each insured has in the time period - i, with $i = 0,1,2, \ldots, t$ and $\sum_{i=1}^{t} k_i = K$, shows the number of claim frequencies in t-year. The distribution of $k_1, k_2, \ldots, k_t$ is Poisson distribution with parameters λ and the probability mass function is

$$p(k_1, k_2, \ldots, k_t|\lambda) = \frac{e^{-\lambda t}\lambda^K}{\prod_{i=1}^{t} k_i!}. \tag{3.3}$$

Bayesian approach can be used to estimate the parameter of claim frequency. The posterior distribution of the claim frequency parameter is obtained by the combined probability density function of several claims $k_1, k_2, \ldots k_t$for t-year and the probability density function of λ. So that the posterior parameter of the number of claims is obtained

$$\begin{aligned} U(\lambda|k_1, k_2, \ldots, k_t) &\propto p(k_1, k_2, \ldots, k_t|\lambda)u(\lambda) \\ &\propto e^{-\lambda t}\lambda^K e^{-\theta\lambda} \\ &\propto e^{-\lambda(\theta+t)}\lambda^K \end{aligned}$$

so that

$$U(\lambda|k_1, k_2, \ldots, k_t) = \frac{(\theta + t)^{K+1}}{\Gamma(K+1)}\ e^{-\lambda(\theta+t)}\lambda^K\ , \lambda > 0. \tag{3.4}$$

The parameter estimation is by minimizing the expected value of the quadratic loss function. Therefore, the Bayesian solution for parameter estimation is equal to the expected value of the number of claims $k_1, , k_2 \ldots, k_t$so that:

$$\begin{aligned} \bar{\lambda}_{t+1} &= E\left(\bar{\lambda}\middle|K\right) \\ &= \bar{\lambda}\frac{K+1}{t\bar{\lambda}+1}. \end{aligned} \tag{3.5}$$

Suppose that the initial premium is $t = 0$denoted by p_0, so the premium that must be paid by the insured to the insurance company in the year $t + 1$is as follows:

$$p_{t+1} = p_0\left(\frac{K+1}{t\bar{\lambda}+1}\right). \tag{3.6}$$

3.2 Premium Modeling of Optimal *Malus Bonus System*

Bonus malus system is one of the insurance systems in which the determination of the premium takes into account the data on the frequency of claims and the amounts of claims submitted by each insured. According to [10] the distribution of claim magnitudes is modeled by the Weibull distribution which is a mixed distribution of exponentials that has a density function:

$$f(x|\theta) = \theta e^{-\theta x} \tag{3.7}$$

and the cumulative distribution function that is

$$f(x|\theta) = 1 - e^{-\theta x} \tag{3.8}$$

Parameters θ are Levy distributed with parameters c, with the probability density function as follows:

$$u(\theta) = \frac{c}{2\sqrt{\pi\theta^3}} \exp\left(-\frac{c^2}{4\theta}\right) \quad ; c > 0, \theta > 0. \tag{3.9}$$

The marginal distribution of X can be calculated using the equation $f(x) = \int_0^\infty f(x|\theta)u(\theta)d\theta$ that forms the probability density function for the Weibull distribution:

$$f(x) = \frac{1}{2} c x^{-\frac{1}{2}} e^{c\sqrt{x}} \;; 0 < x < \infty \tag{3.10}$$

so that the cumulative distribution function for the Weibull distribution is:

$$F(x) = 1 - e^{-c\sqrt{x}} \quad ; x \geq 0, c > 0. \tag{3.11}$$

For example, x_i by $i = 1,2, \ldots, K$, stating the amounts of claims i, then the total amount of claims for a policyholder during $t$the period is $\sum_{i=1}^{K} x_i$. Posterior distribution for parameters θ can be obtained using the Bayes approach as follows

$$\begin{aligned} U(\theta|x_1, x_2, \ldots x_K) &= \frac{f(x_1, x_2, \ldots, x_n|\theta)u(\theta)}{\int f(x_1, x_2, \ldots, x_n|\theta)u(\theta)d\theta} \\ &= \frac{\theta^{K-\frac{3}{2}} e^{-\left(\frac{c^2}{4\theta} + \theta\left(\sum_{i=1}^{K} x_i\right)\right)}}{\int_0^\infty \theta^{K-\frac{3}{2}} e^{-\left(\frac{c^2}{4\theta} + \theta\left(\sum_{i=1}^{K} x_i\right)\right)} d\theta} \end{aligned} \tag{3.12}$$

by modifying equation (3.12) it is obtained

$U(\theta|x_1, x_2, \ldots, x_n)$

$$= \frac{\left(\frac{c}{2\sqrt{\sum_{i=1}^{K} x_i}}\right)^{-\left(K-\frac{1}{2}\right)} \theta^{K-\frac{3}{2}} e^{-\left(\frac{c^2}{4\theta} + \theta\left(\sum_{i=1}^{K} x_i\right)\right)}}{\int_0^\infty \left(\frac{2\sqrt{\sum_{i=1}^{K} x_i}}{c}\right)^{K-\frac{3}{2}} e^{-\frac{c\sqrt{\sum_{i=1}^{K} x_i}}{2}\left(\frac{2\sqrt{\sum_{i=1}^{K} x_i\theta}}{c} + \frac{c}{2\sqrt{\sum_{i=1}^{K} x_i\theta}}\right)} d\left(\frac{2\sqrt{\left(\sum_{i=1}^{K} x_i\right)\theta}}{c}\right)} \tag{3.13}$$

then for the denominator of the integral part of the equation (3.13) it can be solved using the *Bassel function* to be

$$B_v(x) = \frac{1}{2}\int_0^\infty e^{-\frac{x}{2}\left(y+\frac{1}{y}\right)} y^{v-1} dy. \tag{3.14}$$

So that the posterior distribution for the size of the claim is:

$$U(\theta) = \frac{\left(\frac{c}{2\sqrt{\sum_{i=1}^{K} x_i}}\right)^{-\left(K-\frac{1}{2}\right)} \theta^{K-\frac{3}{2}} e^{-\left(\frac{c^2}{4\theta}+\theta\left(\sum_{i=1}^{K} x_i\right)\right)}}{2B_{K-\frac{1}{2}}\left(c\sqrt{\sum_{i=1}^{K} x_i}\right)}. \tag{3.15}$$

Estimating the average amount of loss for the next period $\hat{\theta}_{t+1}$using the quadratic loss function, if it is known that the amount of loss $x_1, x_2, \dots, x_k$over a period of time $t$and Kclaims are

$$\hat{\theta}_{t+1} = \frac{2\sqrt{\sum_{i=1}^{K} x_i}}{c}\left(\frac{B_{K-\frac{3}{2}}\left(c\sqrt{\sum_{i=1}^{K} x_i}\right)}{B_{K-\frac{1}{2}}\left(c\sqrt{\sum_{i=1}^{K} x_i}\right)}\right). \tag{3.16}$$

Calculation of optimal *bonus malus* premiums takes into account data on the frequency of claims $k_1, k_2, \dots, k_t$but also data on the amounts of claims $x_1, x_2, \dots, x_k$, in order to obtain the premium to be paid by the insured as follows:

$$p_{t+1} = \left(\frac{K+1}{t+\theta}\right)\left(\frac{2\sqrt{\sum_{i=1}^{K} x_i}}{c}\right)\left(\frac{B_{K-\frac{3}{2}}\left(c\sqrt{\sum_{i=1}^{K} x_i}\right)}{B_{K-\frac{1}{2}}\left(c\sqrt{\sum_{i=1}^{K} x_i}\right)}\right) \tag{3.17}$$

From the equation (3.17)it can be seen that the calculation of the premium is not defined when the policyholder does not submit a claim. So it is necessary to re-analyze the $(K = 0)$.following cases $K = 0$,

$$Premi_{K=0} = \frac{1}{t+\theta}\left(\frac{2}{c^2}\right). \tag{3.18}$$

3.3 Application of the Premium Calculation Model in the *Bonus Malus System*

The simulation data for calculating the *bonus malus system* uses hypothetical data generated with the help of *R studio software*. Meanwhile, the table for calculating the premium for the *bonus malus system* when not submitting a claim $(K = 0)$ and during the period $t + 1$is obtained using the help of Microsoft Excel. Policy data in the form of the frequency of claims and the amount of claims obtained there are 1000 policyholder data obtained in table 3.1

Table 3.1 Data on Policyholder Claim Frequency

Claim frequency (k)	The number of policyholders(n_k)	$(k \times n_k)$
0	599	0
1	285	285
2	95	190
3	18	54
4	3	12
Amount	1000	541

From table 3.1 it can be stated that there were 599 policyholders who did not file claims, 285 policyholders who filed claims for one claim, 95 policyholders who filed claims for two claims, 18 policyholders who filed claims for three times and 3 policyholders who claim four times during their insurance period.

3.4 Implementation of the Premium Calculation of the Classic *Bonus Malus System*

Determination of the premium by using the classical *bonus malus system* is obtained by the component of the claim frequency data. Claim frequency data in this study is assumed to use a geometric distribution. In the data used in the premium calculation using the *bonus malus system*, there are 1000 policyholders. Parameter estimation θ from the claim frequency data is $0{,}5$ and for the value $\bar{\lambda}$ obtained the average of the data obtained is $0{,}541$. Calculation of the premium using the classical *bonus malus system* can be obtained by using equation (3.6). In table 3.2 it can be seen that the premium calculation with the optimal *bonus malus system*, for example, for the initial premium or$p_0 = Rp\ 350.000$.

Table 3.2 Classic *Bonus Malus* System Premium

year (t)	Number of claims (K)				
	0	1	2	3	4
0	350000				
1	227125	454250	681375	908500	1135626
2	168107	336215	504322	672430	840537
3	133435	266870	400304	533739	667174
4	110619	221238	331858	442477	553097
5	94466	188933	283400	377867	472334
6	82430	164861	247291	329722	412152
7	73114	146229	219344	292458	365573

From table 3.2, if the initial premium for the insured is for example Rp. 350,000 then the insured will get a reduction in the premium rate to Rp. 227,125 or in other words, you will get *a bonus* of 35% of the initial premium provided that the insured does not make a claim in the previous year (provided that if the insured extends his insurance policy). If the same insured makes one claim in the first year, then the insured will get an increase in the premium rate of IDR 454,250 or will pay a *malus* of 22% of the initial premium.

3.5 Optimal *Bonus Malus* System Premium Calculation

The components of claim frequency and claim size can be used to calculate optimal *bonus malus premiums* with claim size data in this study assumed to use the Weibull distribution and claim frequency data use the Geometric distribution. Parameter estimation θin the claim frequency data is assumed to be 0,5. As for the parameter $c$estimation value the Weibull distribution on the claim amount data can be calculated based on the equation (3.10), namely:

$$\hat{c} = \frac{n}{\sum_{i=1}^{n} (x_i)^{\frac{1}{2}}} = \frac{401}{139007{,}6527} = 0{,}002.$$

Calculation of the premium paid by each insured in the period $t+1$with the optimal *bonus malus* system can be obtained using the equation (3.17). Meanwhile, to calculate the premium for each insured who does not make a claim, $(K = 0)$it can be calculated using the equation (3.18).in table 4.3. It can be seen that the premium calculation for the optimal malus bonus system with a total claim size in 1 year $\sum_{i=1}^{K} x_i = Rp\ 3.500.000$.

Table 3 .3 Premium *Bonus Malus System* Optimal

year (t)	Number of claims (K)				
	0	1	2	3	4
0	1000000				
1	333333	2494438	3741657	4988877	6236096
2	200000	1496663	2244994	2993326	3741657
3	142857	1069045	1603567	2138090	2672612
4	111111	831479	1247219	1662959	2078699
5	90909	680301	1020452	1360603	1700753
6	76923	575639	863459	1151279	1439099
7	66666	498887	748331	997775	1247219

The explanation from table 3.3 is that if the insured in the initial period did not have an accident or did not make a claim, the initial premium was Rp. 1,000,000 and will get a reduction in premium rates in the first year, namely to Rp. 333,333 or in other words will get *a bonus* of 66%. Then if the insured makes a claim with two claims in the second year, the insured will get an increase in the premium rate to Rp. 1,603,567 or in other words, you have to pay *a malus* of 37%the initial premium. In the optimal calculation of the premium *bonus malus system*, the greater the amounts of claims submitted by the insured, the greater the premium to be paid. So that each insured will pay a different risk premium depending on the number of claims and the size of the loss.

IV. CONCLUSION

Classics bonus malus system to determine the calculation of the premium paid by the insured does not depend on the size of the claim, so that the calculation of the premium results in unfair results. So that the premium amount can be calculated based on the bonus malus system if it only depends on the frequency of claim. *Optimal bonus malus system* to determine the calculation of the premium paid by the insured depends on the frequency of claims and the size of the claim, so that it is fair for all insureds who renew in the following year (renew). So that the amount of premium can be calculated based on the optimal bonus malus system which does not only depend on the frequency of claims, but also on the amount of claims.

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