III. KESIMPULAN

Pada Lema 3, baru diperoleh syarat perlu dari $a^+(a^+)^* = (a^+)^*a^+$ untuk $a \in R^+$, dan perlu dibangun syarat cukupya. Dengan memperoleh syarat perlu dan cukup $a^+(a^+)^* = (a^+)^*a^+$ untuk $a \in R^+$, akan dapat dibangun syarat perlu dan cukup $a \in R^+$ adalah normal, dengan melibatkan Teorema 1.

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