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FREE CONVECTION FLOW OVER A HORIZONTAL POROUS FLAT PLATE WITH THE EFFECT OF MAGNETOHYDRODYNAMICS

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Abstract. We are interested in study of the velocity (f') and temperature (θ) profiles for fluid flow over the surface of porous flat plate with the effect of magnetohydrodynamics. The dimensional equations are first transformed into the non-dimensional equations. Then, we transform the non-dimensional equations into the similar equations using stream functions. The numerical results are based on the discretization of similar equations using the finite difference method of Keller-Box. Based on the numerical results, the velocity profiles (f') decrease when the viscoelastic parameter (K), Prandtl number (P_r), magnetic parameter (M), porosity parameter (P) are increased. Moreover, the temperature profile (θ) is increased when the viscoelastic parameter (K) and magnetic parameter (M) are increased. However, the temperature profile (θ) decreases when Prandtl number (P_r) and porosity (P) are increased.
Keywords: magnetohydrodynamics, porosity, viscoelastic fluid, Prandtl number.

I. INTRODUCTION

Non-Newtonian fluids research has grew-up so fast because of more application in industrial fields such as in petroleum production, wire drawing, paper production etc. Walters-B was one of viscoelastic fluid model which was first developed by Walters [3] and its future research was studied in [9]. The result has shown that different choices of the measure of strain correspond different theories of finite linear viscoelasticity.

Much work has been done in order to understand the effect of velocity profile and heat transfer in viscoelastic fluids [10, 11, 14, 18]. Heat and velocity analysis fluid flow past over a flat plate had much attention for researcher because many branches of science and technology employed this technique as in the study of [1, 2, 5, 12]. In most of the studies, the effects of magnetohydrodynamics in a fluid flow became interested because of its application in engineering [4, 6, 13, 16, 17, 19].

Kayvan [8] has presented that all parameters such as the Reynolds number, the Weissenberg number, and the magnetic number has a profound effect on the velocity profiles. Kasim [7] has studied magnetohydrodynamic flow of viscoelastic fluid past over a flat plate for the steady state and incompressible, that was solved numerically by Box-Keller method.

In this research, the profile of velocity (f') and of temperature (θ) for free convection flow in viscoelastic fluid past over a porous flat plate with the effect of magnetohydrodynamics is studied and solved numerically using Keller-Box method. This present paper is the modified version of researches studied in [12] and [14], where the porosity flat plate is considerable in this paper.

The rest of paper is organized as follows. In Section 2, we present the mathematical model of fluid flow over a horizontal porous flat plate, and the boundary conditions. Section 3 is devoted to establish the numerical results, where the algorithm of Keller-Box and its discretization are presented in this section. In Section 4, we establish the summaries for all numerical results of discretization results obtained from the previous section.

II. MATHEMATICAL MODEL

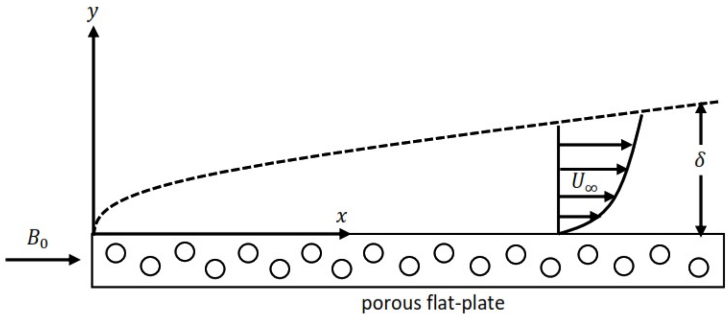

Figure 1. Schematic view of porous flat plate

This problem considered steady two-dimensional flow with constant velocity U_w to the free stream velocity U_∞, as shown in Figure 1, where the x-axis extends parallel to the plate and y-axis extends upwards normal to the plate. The type of tensor that is used in the momentum equation is Walters'B fluid [9, 15]. Thermodynamics conservation law, Newton's second law, and mass conservation are used to construct the following modified mathematical model studied in [20].

$$\begin{aligned} &\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \\ &u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 \\ &u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\frac{\mu_0}{\rho}\left(\frac{\partial^2 u}{\partial y^2}\right)-\frac{k_0}{\rho}\left(u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right) \\ &\qquad -\frac{k_0}{\rho}\left(\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}\right)-\frac{1}{\rho}\sigma u B_0^2-\frac{\nu}{K}u \end{aligned} \tag{1}$$

with the boundary condition

$$\begin{aligned} &u=U_x, \quad v=V_x, \quad T=T_w \quad \text{at} \quad y=0 \\ &u=U_\infty, \quad \frac{\partial u}{\partial y}=0, \quad T=T_\infty \quad \text{at} \quad y\to 0 \end{aligned} \tag{2}$$

We use stream function of ψ to analyze the profile of velocity and temperature on the boundary layer. So, the velocity components can be written as

$$u=\frac{\partial \psi}{\partial y}, \quad v=-\frac{\partial \psi}{\partial x} \tag{3}$$

which can be transformed into

$$\psi = U_\infty x v \sqrt{2} f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \eta = \sqrt{\frac{U_\infty}{2xv}} y \tag{4}$$

By substituting (3) and (4) into (1), the non-dimensional stream function equations can be obtained

$$\begin{aligned} &\frac{1}{P_r}\theta'' + f\theta' + E_c(f'')^2 = 0 \\ &f''' + ff'' + \frac{K}{2}(ff'''' + f'f''' - (f'')^2) - Mf' - Pf' = 0 \end{aligned} \tag{5}$$

with the boundary conditions

$$\begin{aligned} &f(0) = f_w, \quad f'(0) = \lambda_m, \quad f'(\infty) = 1.5 \\ &f''(0) = 0, \quad \theta(0) = 1, \quad \theta(\infty) = 0 \end{aligned} \tag{6}$$

where the viscoelastic parameter (K) , moving parameter (λ_m), porosity (P), Euckert number (E_c), and magnetic parameter (M) defined as

$$K = \frac{k_0 U_\infty}{\rho v}, \quad \lambda_m = \frac{U_w}{U_\infty}, \quad P = \sqrt{\frac{k_0}{g\beta T_w}\frac{v}{K}}, \quad E_c = \frac{U_\infty^3}{c_p(T_w - T_\infty)}, \quad M = \frac{k_0 U_\infty}{\rho v} \tag{7}$$

III. NUMERICAL RESULTS AND DISCUSSION

The non-dimensional stream function equations are solved by Keller-Box method. We simulate the discretization using the software MATLAB 2010a, then we get the velocity profile (f') and the temperature profile (θ) for the fluid flow over the porous flat plate with the effect of magnetohydrodynamic. Moreover, the algorithm of Keller-Box for this case can be stated as follows

Algorithm 1. Keller-Box method

input : initializing the value of P, M, K, P_r, boundary layer thickness y, step size h

procedure

$\delta f_0 = 0, \delta u_0 = 0, \delta s_0 = 0, \delta u_N = 0, \delta v_N = 0, \delta s_N = 0$

$f(j,1) = 1.5 * \left(y(j) + e^{-y(j)} - 1\right)$

$u(j,1) = 1.5 * \left(1 - e^{-y(j)}\right)$

$v(j,1) = 1.5 * e^{-y(j)}$

$w(j,1) = -1.5 * e^{-y(j)}$

$s(j,1) = 1 - \left(\frac{y(j)}{y(N)}\right)$

$t(j,1) = -\frac{1}{y(N)}$

for $j = 1$ ***to*** N ***do***

if $(j = 1)$ ***do***

$[\alpha_1] = [A_1]$

$[\alpha_j][\Gamma_j] = [C_j]$

$[\alpha_1][w_1] = [r_1]$
else if $(j = N)$ ***do***
$[\alpha_j] = [A_j] - [B_j][\Gamma_{j-1}]$
$[\alpha_j][w_j] = [r_j] - [B_j][w_{j-1}]$
else
$[\alpha_j] = [A_j] - [B_j][\Gamma_{j-1}]$
$[\alpha_j][\Gamma_j] = [C_j]$
$[\alpha_j][w_j] = [r_j] - [B_j][w_{j-1}]$
end if
end for j *% step for update value of* δ *using backward sweep*
for $j = 1$ ***to*** N ***do***
if $(j = N)$
$[\delta_j] = [w_j]$
else
$[\delta_j] = [w_j] - [\Gamma_j][\delta_{j+1}]$
end if
end for j *% step for update value of* f, u, v, w, s *and* t
for $j = 1$ ***to*** $N - 1$ ***do***
$f(j+1) = f(j) + \delta f(j);$ $\quad u(j+1) = u(j) + \delta u(j)$
$v(j+1) = v(j) + \delta v(j);$ $\quad w(j+1) = w(j) + \delta w(j)$
$s(j+1) = s(j) + \delta s(j);$ $\quad t(j+1) = t(j) + \delta t(j)$
end for j
end procedure
output : profiles of velocity (f'), profile of temperature (θ)

The stencil of Keller-Box for the steady-stated is in the following Figure 2

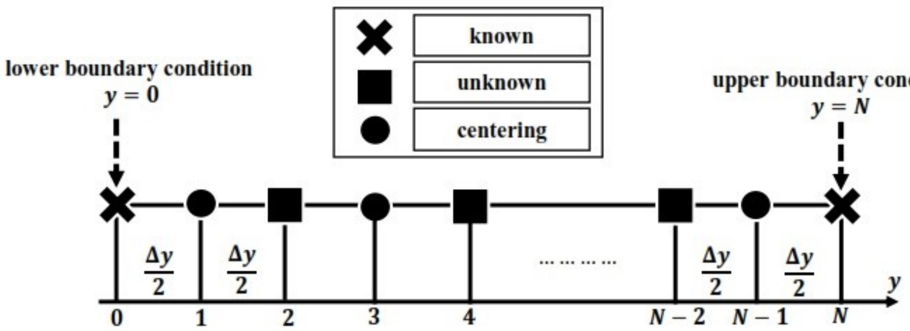

Figure 2. Stencil of Keller-Box in steady state

Figure 2 shows the stencil of Keller-Box for the steady state with step size of $\Delta y/2$, the lower boundary condition $y = 0$ and upper boundary condition $y = N$. Moreover, y is the thickness of boundary layer of shear stress between viscoelastic fluid and the surface of porous flat plate. Based on these conditions, the stream function equations are first changed into first order as stated as follows

$$f' = u, \qquad u' = v, \qquad v' = w, \qquad s' = t$$

$$w + fv + \frac{K}{2}(uw + fw' - v^2) - Mu - Pu = 0, \qquad \frac{1}{P_r}t' + ft + E_c v^2 = 0 \qquad (8)$$

Equations (8) are stated in block matrix and then solved using LU decomposition. The numerical results of velocity profile (f') and temperature profile (θ) are as follows.

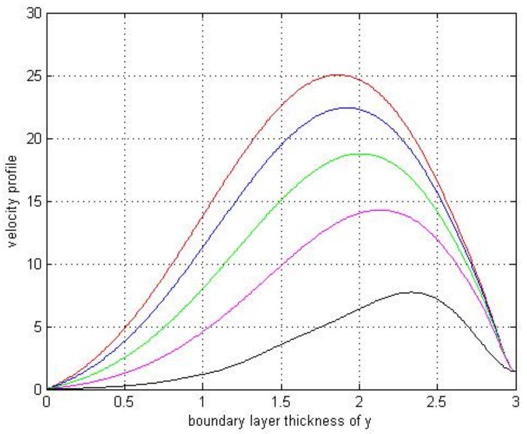

Figure 3. Velocity profile (f') with the thickness of boundary layer (y) for various values of magnetohydrodynamics (M)

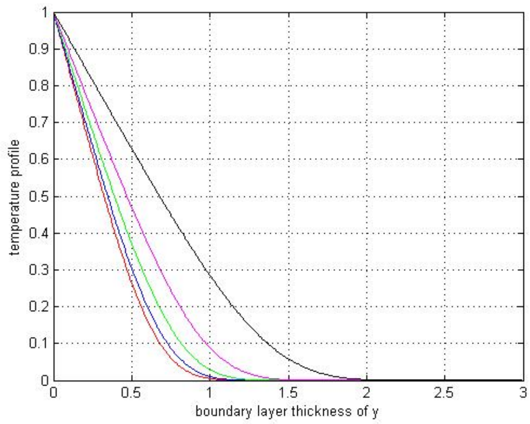

Figure 4. Temperature profile (θ) with the thickness of boundary layer (y) for various values of magnetohydrodynamics (M)

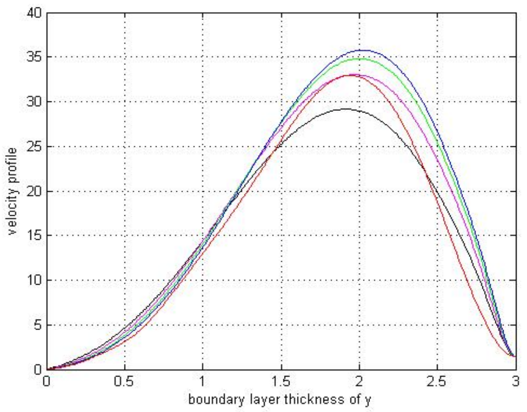

Figure 5. Velocity profile (f') with the thickness of boundary layer (y) for various values of viscoelastic (K)

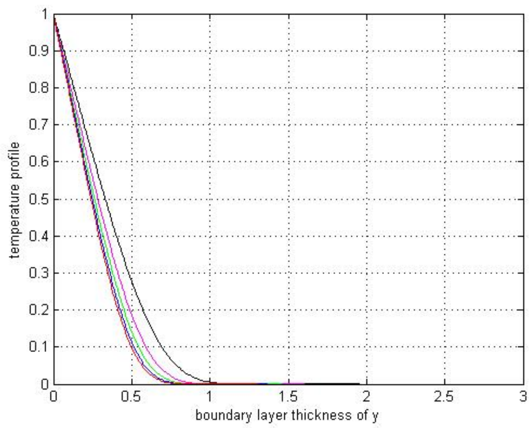

Figure 6. Temperature profile (θ) with the thickness of boundary layer (y) for various values of viscoelastic (K)

The existence of a transverse magnetic field to an electrically conducting fluid gives rise to a type force, called as Lorentz force. This force has the tendency to slow down the motion of the fluid. The result qualitatively agrees with the expectations, since magnetic field gives force on the free convective flow which decreases the motion of the fluid as shown in Figure 3. The presence of transverse magnetic field produces the Lorentz force. As the Lorentz force increases, the fluid exhibits a resistance to this force by increasing the friction between its layers. This resistance appears as an increase in the temperature, the temperature profile increases when the magnetic parameter increases as shown in Figure 4. It is observed from Figure 5 and Figure 6 show that the increases of the viscoelastic parameter increases the temperature of the fluids but decreases the velocity profile.

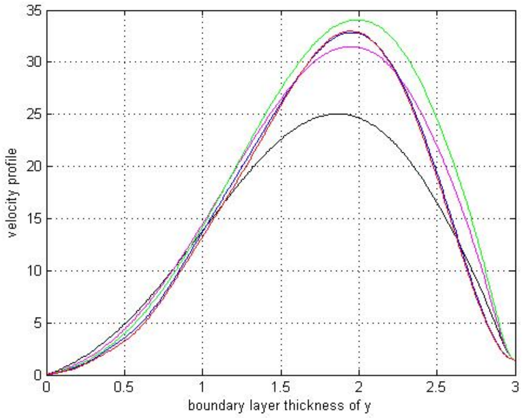

Figure 7. Velocity profile (f') with the thickness of boundary layer (y) for various values of Parndtl (P_r)

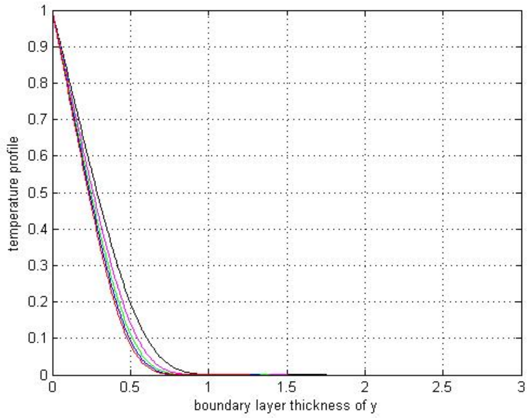

Figure 8. Temperature profile (θ) with the thickness of boundary layer (y) for various values of Prandtl (P_r)

It is clear from Figure 7 that the velocity profiles decrease for increasing the Prandtl number. As the Prandtl number increases, viscous forces tend to suppress the buoyancy force which decreases the fluid velocity in the boundary layer. Temperature profile decreases when Prandtl number increases as in Figure 8.

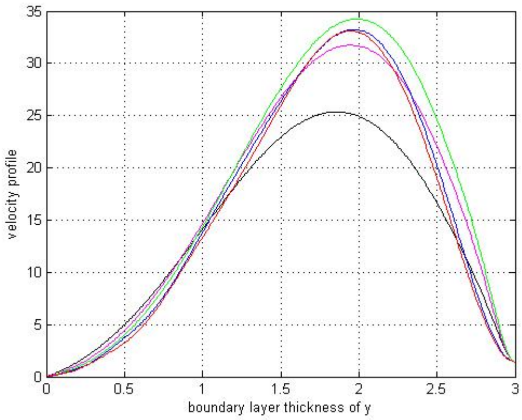

Figure 9. Velocity profile (f') with the thickness of boundary layer (y) for various values of porosity (P)

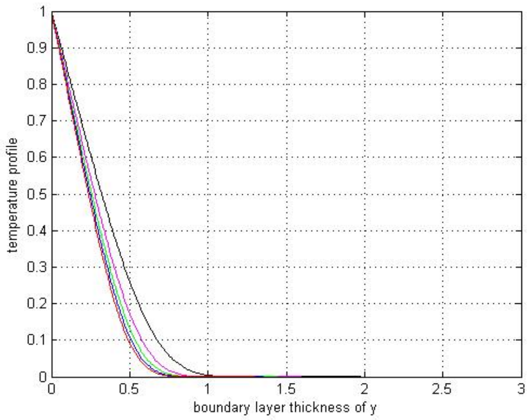

Figure 10. Velocity profile (f') with the thickness of boundary layer (y) for various values of porosity (P)

Figure 9 describes about the profile graphic of fluid velocity with variation of porosity parameters (P). From the simulation results shown that the smaller the flat plate porosity values, resulting in the flow velocity is increased. Conversely, if value the porosity of the larger flat plate, then the velocity of fluid flow is decreased. So, it can be concluded that the value the parameter porosity (P) is inversely proportional to the flow rate through the porous surface of flat plate. Figure 10 describes about the profile of fluid temperature with variation of porosity parameters (P). From the simulation results, it shows that the smaller the flat plate porosity values, resulting in the temperature of the flow is decreased. Conversely, if the value of the porosity of flat plate is greater, then the temperature of the fluid flow increases. This is due to the influence of the porosity parameters given. So, it can be concluded that the porosity parameter is proportional to the temperature of the flow through the porous surface of flat plate.

IV. CONCLUSION

We have examined the influence of variable magnetic on viscoelastic fluid flow over a porous flat plate. The Keller-Box is used to solve the problem and the numerical results are presented to analyze the temperature profile and velocity profile. The following main conclusions can be drawn from the present study: the velocity profiles decrease for the increasing of viscoelastic parameter (K), Prandtl number (P_r), magnetic parameter (M), porosity parameter (P) and the temperature profile increases for increasing of viscoelastic parameter (K), magnetic parameter (M), but temperature profile decrease for increasing of Prandtl number (P_r), and porosity (P).

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