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MATHEMATICAL EXPANSION OF SPECIAL THEORY OF RELATIVITY ONTO ACCELERATIONS

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Abstract. The special theory of relativity (STR) is operationally expanded onto orthogonal accelerations: normal a_N and binormal a_B that complement the instantaneous tangential speed v_T and thus can be structurally extended into operationally complete 4D spacetime without defying the STR. Thus the former classic Lorentz factor, which defines proper time differential $d\tau = \sqrt{1-\frac{v^2}{c^2}}\,dt$ can be expanded onto $d\tau = \frac{1}{c}\sqrt{c^2 - v_T^2 - v_N^2 - v_B^2}\,dt = \frac{1}{c}\sqrt{c^2 - v_T^2 - (t_N a_N)^2 - (t_B a_B)^2}\,dt$ within a trihedron moving in the Frenet frame (**T,N,B)**. Since the tangential speed v_T which was formerly assumed as being always constant, expands onto effective normal and binormal speeds ensuing from the normal and binormal accelerations, the expanded formula conforms to the former Lorentz factor. The obvious though previously overlooked fact that in order to change an initial speed one must apply accelerations (or decelerations, which are reverse accelerations), made the Einstein's STR incomplete for it did not apply to nongravitational selfpropelled motion. Like a toy car lacking accelerator pedal, the STR could drive nowhere. Yet some scientists were teaching for over 115 years that the incomplete STR is just fine by pretending that gravity should take care of the absent accelerator. But gravity could not drive cars along even surface of earth. Gravity could only pull the car down along with the physics that peddled the nonsense while suppressing attempts at its rectification. The expanded formula neither defies the STR nor the general theory of relativity (GTR) which is just radial theory of gravitation. In fact, the expanded formula complements the STR and thus it supplements the GTR too. The famous Hafele-Keating experiments virtually confirmed the validity of the expanded formula proposed here.

Keywords: Expanded special relativity, special-relativistic accelerations, univocal special relativity

I. INTRODUCTION

Some remarks are so impressive that one may feel compelled to reevaluate afresh everything we learned heretofore. One of such profound remarks to me is that if physical laws evolve in time, then time must play a preferred role in the world; it cannot be just another dimension and it cannot just disappear in a timeless block universe or "wavefunction of the universe" [1]. In conjunction with the assertion that superspace is not a manifold but a stratified union of manifolds and spacetime is a sheaf of geodesics in the superspace [2] one may wonder what the abstract mathematical time really is. Other profoundly innovative to me assertions were that mass and electrostatic potential, which were known as scalars, are found to be vectors in time and that scalar potential present in 4D Lorentz equation should be [represented by] a contravariant vector in time [3] and that 4D world is spherical with respect to space but

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