

Quantum Computing in Supply Chain Management State of the Art and Research Directions

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Abstract Quantum computing is the most promising computational advance of the coming decade for solving the most challenging problems in supply chain management and logistics. This paper reviews the state-of-the-art of quantum computing and provides directions for future research. First, general concepts relevant to quantum computers and quantum computing are introduced. Second, the dominating quantum technologies are presented. Third, the quantum industry is analyzed, and recent applications in different fields of supply chain management and logistics are illustrated. Fourth, directions for future research are given. We hope this review to educate and inspire the use of quantum computing in the fields of optimization, artificial intelligence, and machine learning for supply chain and logistics.

Keywords Quantum Computing; Review; Supply Chain Technology

INTRODUCTION

The concept of quantum computing (QC) was introduced in the early 1980s (Feynmann, 1982), Beniof, 1980 using quantum effects while building a new type of computer hardware, the quantum computer. To use quantum computers, new types of algorithms, quantum algorithms, were developed. The foundations date back to David Deutsch in 1985 with the Deutsch-Algorithm (Deutsch 1985), Peter W. Shor in 1994 with the Shor-Algorithm [Shor, 2002)], and Lov K. Gover in 1996 with the Gover-Algorithm [Grober, 2021], and others (). They are examples of quantum algorithms that solve computationally intense, NP-hard, and P-hard problems that outperform classical algorithms (Ball, 2020).

Simply put, QC differs from classical (binary) computers by taking advantage of properties unique to quantum mechanics, making it possible to probe more possibilities simultaneously. This is especially useful for the large-scale combinatorial problems that are typical for supply chain and

logistics. Most optimization problems common in supply chain, such as planning, scheduling, decision making, data analytics, logistics, and many other fields (Pardalos, 1987) are getting computationally expensive while increasing e.g., the search space or the number of constraints. Nowadays, quantum (heuristic) algorithms have been proposed with promising performance. For instance, back in 2019, Google claimed to have reached 'quantum supremacy' [Murgia and Water, 2019]. Series of quantum operations were performed in 200 seconds which would take a classical supercomputer about 10'000 years to do (following Google) or two and a half days (following IBM (Ray,2021). Either way, it is widely accepted that QC can find answers within the relevant time where classical supercomputers fail today.

There are two prominent quantum technologies, the quantum gate-based and the quantum annealing-based models (Preskill, 2018), as well as hybrid settings. While quantum gate computers allow programming with logical gates like classical computers, quantum annealers work differently (Johnson et al, 2014; Smolin and Smith, 2014). In what follows, this review focuses more on quantum annealers that solve problems without the need to understand the underlying quantum mechanical effects. This is because unlike quantum annealers, gate-based quantum computers do not have sufficient qubits to solve today's real-world problems (Leprince-Ringuet, 2021).

This paper reviews important scientific and specialist publications from January 2016 to August 2021 in the field of QC to explore the application potential of QC and identify promising avenues of investigation for the supply chain and logistics researchers. The review is organized as follows: Section 2 reviews quantum computing, including quantum effects, quantum computers, and the state of the QC industry. Section 3 describes current frameworks, and applications in various supply chain settings. Section 4 deducts research directions for applied quantum research in supply chain and logistics.

Quantum Computing

Classical information theory is a mathematical theory of processing, storing, and transmitting information. Quantum information theory investigates how quantum mechanical systems can perform similar tasks (Datta, 2006) by applying quantum mechanics effects, which are usable with different kinds of QC, and are made available to business users and researchers through different means.

Quantum Effects

There exist four QC key effects: superposition, interference, entanglement, and tunnelling. The first effect of QC is superposition. The fundamental unit of information in classical computers is referred to as classical bit (also mentioned as cbit) and is in either of two discrete states, "0" or "1". In contrast, the fundamental units of QC are called quantum bits (qubit). Like the bit, a qubit is based on two basic states, but is a two-state quantum-mechanical system. The quantum state of a qubit exists in a superposition of both basic states simultaneously. Therefore, qubits enable

the exploration of richer sets of states in an extended solution space, as shown in Figure 1. As one qubit represents a superposition of two states, two qubits represent four states, three cubits eight and so forth (see Table 1). Thus, the simultaneously

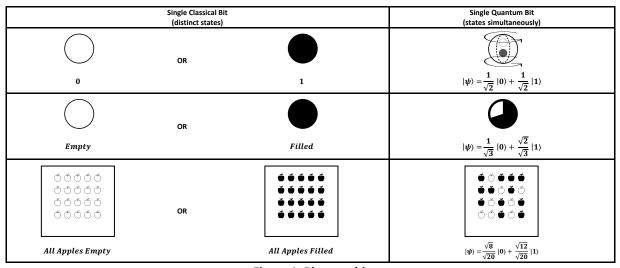


Figure 1. Bit vs. qubit (own figure based on Hidary, 2019)

Figure 1 explains the increased information density due to the unique possibility of superpositions in a qubit compared to a bit. Each row is an example of information represented by a single bit compared to a single qubit. Therefore, a single qubit can represent much more information than a single bit. For example, the first row shows the two distinct states of "0" or "1" of a single bit and in a single qubit in a superposition state of "0" and "1" simultaneously. The second row shows the two distinctive states of a bit again, while a single qubit can also represent 1/3 as empty and 2/3 as filled. The same goes for the third row. If a single bit represents the state of 20 apples, all apples are either empty or filled. In contrast, representing the state of 20 apples as a single qubit in a superposition enables to have, for example, 8/20 empty and 12/20 filled apples simultaneously.

A bit is often represented as a factor of two elements $(\alpha, \beta)^T$ where α stands for "0" and β stands for "1". Therefore, the state "0" is represented as $(1, 0)^T$ and "1" as $(0, 1)^T$. Qubits are a linear combination of two possible states of "0" and "1". A qubit is any combination $(\alpha, \beta)^T$ where α and β are complex numbers that satisfy the normalization condition $|\alpha|^2 + |\beta|^2 = 1$ The normalization is necessary to interpret $|\alpha|^2$ and $|\beta|^2$ as probabilities for measure outcomes of the two primary states "0" and "1". Eq. (1) shows the quantum states as a quantum states system in Dirac Notation (Hidary, 2019)

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle, where |0\rangle = \begin{bmatrix} 1\\0 \end{bmatrix}, |1\rangle = \begin{bmatrix} 0\\1 \end{bmatrix}, and |\alpha|^2 + |\beta|^2 = 1$$
(1)

The second effect is quantum interference, which biases the qubit towards a desired state or set of states. This effect can be explained by a marked coin that tends towards the head or the tail. The third effect, entanglement, is the possibility of two or more quantum mechanical systems becoming a single unique system. When qubits are entangled, they form a system where the quantum state of each qubit in the system cannot be described independently. Thus, any operation applied to one of the entangled qubits correlates to the states of the others which enables the prediction of possible states of one qubit by measuring the other. The fourth effect is quantum tunnelling, in which particles behave like waves and propagate through a potential barrier. This wave weakens when it hits the barrier, but in quantum mechanics there is the probability that the particle will pass the barrier on the other side. This effect is especially interesting for quantum annealers and allows them not to 'get stuck' in local optima. Together, the above unique properties of QC increase the density of information and enable QC to manipulate many combinations of states at once, and thus to find solutions by simultaneously exploring multiple states in search spaces in concurrent applications. Table 1 illustrates how qubits and classical bits compare.

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Qubits	2	3	10	16	20	30	35	100	280
bits required to	512	1024	16	1	17	16	550	more than	more than
represent an	bits	bits	KB	MB	MB	GB	GB	atoms on	atoms in the
entangled state								earth	universe

Table 1. Air Transport of Goods By Country	y
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Quantum Computers

There exist two types of QC: quantum gate-based and annealer-type quantum computers, as well as hybrid variations.

The quantum gate-based model resembles classical computers. Individual qubits are directly programmable with quantum gates, like logical gates in conventional digital circuits. A quantum gate-based processor is thus a universal programmable calculator that converts inputs of qubits into outputs of qubits (Matthews, 2021). Like AND, OR, and NOT gates, which manipulate bits in logical circuits, quantum gates transform qubits in quantum circuits. The processors that implement quantum gate-based models with more than 50 fully connected qubits are called noisy-intermediate scale quantum (NISQ) devices. The lower limit of about 50 qubits is considered the threshold above which quantum devices can no longer be reasonably simulated by traditional supercomputers (Guerreschi and Matsuura, 2019).

Unlike many classical logical gates, a QC can only perform reversible operations due to the nature of quantum mechanics. For example, setting an individual bit to "1" or "0" like in classical computers is irreversible and invalid in quantum systems. However, it is possible to perform classical computing using only reversible gates and ancilla bits, extra qubits only used for the operation. The advantages of quantum gate computers are the programmability with logical quantum gates (Matthews, 2021). The disadvantages of quantum gate computers today are that they do not have enough qubits to solve many business-relevant problems (Leprince-Ringuet,

2021), and that the qubits in a quantum gate computer need intense error correction (Farhi et al., 2000). However, this will change as QC technology improves and QC with 1'000 qubits enter the market (IBM, 2020; Cho, 2020].

In contrast to the quantum gate-based model, quantum annealing is a metaheuristic approach to solving optimization problems (Johnson, et al., 2011), (Farhi et al., 2000; Das and Chakrabarti, 2009; Smelyanskiye et al., 2012) by optimizing the cost or energy functions of complex systems that emulate quantum fluctuations (Das and Chakrabarti, 2008). Several studies indicate that quantum annealing outperforms traditional simulated annealing (Santoro et al., 2021; Martonak et al., 2002; Baldassi and Zecchina., 2019). Quantum annealing is based on the thermodynamical principle of strive for minimal energy states. In nature, physical systems tend to evolve toward their lowest energy state: objects slide down hills, hot things cool down, and so on. This property makes quantum annealing well-suited to specific optimization problems, where the problem can be formulated as a system that advances to the lowest energy state.

What's more, quantum annealers start their heuristic search with multiple starting points simultaneously and explore the search space in parallel and much faster due to superposition and quantum tunnelling (Wave System Inc., 2021). This is different in classical computing and with classical algorithms, which tend to be either exact methods and must therefore explore the entire search space or use heuristics. The latter typically having to incorporate multiple runs to find global, rather than local optima.

In contrast to the quantum gate-based model, quantum annealers are not programmable, and users cannot modify qubits like in the gate-based model. Instead, quantum annealers take advantage of quantum mechanics by describing a problem as discrete optimization problem formulated as Ising or Quadratic Unconstrained Binary Optimization (QUBO) (D-Wave Systems Inc., 2021). For example, many optimization problems can naturally be expressed as N-hard problem of finding the ground state, or minimum energy configuration, of an Ising model Hamiltonian [12], as depicted in Eq. (2),

$$\min_{q_j,\dots,q_n} \left(\sum_{j=1}^n a_j q_j^z + \sum_{1 \le j < k}^n a_{jk} q_j^z q_k^z \right)$$
(2)

where a_j \in R and pre-defined linear weights, a_jk \in R and pre-defined quadratic coupler weights, and q_j z are Pauli matrices assigning values {1} to spin values { \uparrow, \downarrow }.

Eq. (2) shows the general Hamiltonian minimization of an optimization problem as Ising model with a quantum annealer. The disadvantage of quantum annealers is that they are limited to the above models and are not universally programmable computing machines (D-Wave Systems Inc., 2021).

Quantum Computing Industry and Applications *Quantum Computing Industry* Applied research and development in quantum computing for supply chain is closely linked to some new, and some established companies in the realm of computation. To start mapping the quantum computing industry, Table 2 lists a current set of quantum gate-based computers, while Table 3 Lists potent quantum annealers.

Manufacturer	Name	Qubits	Release	Ref.		
USTC	Jiuzhang	76	2020	(Zhong et al., 2020		
Google	Bristlecone	72	2017	(Kelly, 2018)		
IBM	Hummingbird	65	2020	[21]		
Google	Sycamore	53	2018	(Martinis, 2019)		
IBM	IBM Q 53	53	2019	(Nay and IBM Research, 2019)		
Intel	Tangle Lake	49	2018	(Intel, 2018)		

Table 2. Quantum	Gate Computers Overview
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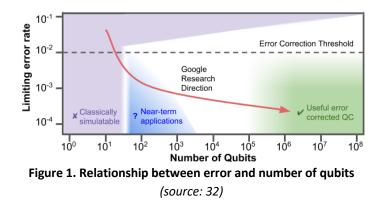
Table 3. Quantum Annealers Overview	
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Manufacturer	Name	Qubits	Release	Ref.
D-Wave	D-Wave 2X	over 1000	2015	-
D-Wave	D-Wave 2000Q	over 2000	2017	(Gibney, 2017)
D-Wave	D-Wave Advantage	over 5000	2020	D-Wave Systems Inc., 2021

Please note that D-Wave systems have more qubits due to specific requirements and architecture. Unlike a quantum gate-model computer, the qubits of the quantum annealer are not fully connected. D-Wave does speak of the physical qubits of their annealers, as opposed to the qubit counts in a gate-based model quantum computer where the number mostly represents the logical programmable qubits.

The performance and possibilities of QC are not only determined by the number of qubits, but also depend on their error-proneness and entanglement potential. A challenge today is to manage the error correction and fault tolerance of a quantum device (Cho, 2020). Many physical entangled qubits are required to safeguard the state of a single logical qubit. According to (Cho, 2020), a QC with 1'000 safeguarded logical qubits requires many million physical qubits. Google has plans to achieve a universal programmable device with 1'000 qubits within the next ten years. IBM plans to have a 1'000 NISQ device by the end of 2023 (IBM, 2020).

Figure 2 illustrates the relationship between error rates and the number of qubits. The arrow depicts Google's purposes for the research and how they hope to access near-term applications on the way to building an error-corrected quantum computer (Kelly, 2018). Before QC became available, many simulators were developed in programming languages, among others C++ (Quantum++) to Java (jQS). An extensive list is available under quantiki.org, 2021.



The quantum computer 'Weber' is currently the only one available by Google, which has 53 qubits because one qubit has a defect (Google, 2021). The list of IBM's quantum processors is long, and the services offered by IBM Quantum System One (IBM, 2019) and IBM Quantum Network (IBM, 2019)2 are unique. Intel invested in quantum hardware research overcoming the challenges in QC scalability to a larger qubit number (Intel, 2020; Petit et al., 2020).

Multiple governments have established programs to explore quantum technologies, such as the UK National Quantum Technologies Programme (Knight and Walmsley, 2019), the Centre for Quantum Technologies in Singapore (Singha Et al, dkk), and a Dutch program for developing quantum computers called QuTech (The Economist, 2015). The European Union announced the Quantum Technology Flagship in 2016, a ten-year and Euro 1 billion megaproject that started in 2018 (Riedel, et al., 2017). In 2018, the United States passed the National Quantum Initiative Act, which provides a budget of US Dollars 1 billion annually for quantum research annually (Raymer and Monroe, 2019).

Quantum research is on the agenda of many top-tier universities. In Switzerland, for instance, ETH Zurich is providing CHF 32 million (Quantencomputer, 2021; Hennemann, 2021) to establish a joint center for the development of quantum computers in May 2021. It aims to advance in the realization of quantum-based computers and build experimental quantum processors and investigate ways to create faster and more reliable qubits (Grimm et al., 2021;)

In industry, leading companies in the private sector such as IBM, D-Wave, Google, Microsoft, and Amazon have made substantial investments and established business models in quantum technology and made QC available as a service. This gives rise to the potential to accelerate the modeling and solution of complex societal, macroeconomic, and environmental problems on a global scale using QC. The key quantum technology providers of 2021 and their products are outlined below, as to facilitate matching with research and applied problems.

IBM Quantum Network, Qiskit. The core technology used by IBM is the quantum gate computer model using NISQ devices (IBM, 2020), which is provided in the IBM Quantum Network, a community of over 140 Fortune Global 500 companies, academic institutions, national labs, and

start-ups (IBM, 2021). This network focuses on three areas: In research, IBM empowers organizations with knowledge to jointly advance in the field of quantum computing. In application development, the Qiskit framework development builds the core. Qiskit is an open-source, modular, and extensible quantum programming framework for application development written in Python. In education, IBM supports and trains academic institutions to acquire and teach the skills necessary to capitalize on QC (IBM, 2021). Since March 2021, IBM is offering the quantum industry's first developer certification (Asfaw et al., 2021) and is promoting Qiskit (IBM, 2021) used in education by many renowned universities, among others the Universities of Cambridge, Melbourne, München, Oxford, Tokyo (IBM, 202. In addition, the IBM Quantum Network comprises huge companies such as Accenture, Boeing, Daimler, ExxonMobil, Samsung, PayPal, and others [IBM Institute. 2021].

D-Wave Quantum Annealer, Ocean. Announced as the first supplier of commercial quantum systems, D-Wave was founded in 1999. It provides the quantum annealing technology via cloud service and an API called Ocean yei you pa8j. Ocean is an open-source framework written in python. The framework implements the computations needed to transform an arbitrarily posed problem into a form suitable for a quantum solver. which is hosted in D-Wave's cloud (D-wave System ink. . The company recently announced a new hybrid solver where users can combine the annealing and gate-model technologyD-Wave also provides a platform for the community of organizations, universities, developers, and researchers (D-Wave Systems Inc, 2021) and thus maintains an education network with renowned universities, among others the Universities of Tokyo, Warsaw, Stanford (D-Wave Systems Inc. 2021)

Additional Industrial Quantum Computing Players. Google offers a QC service to scientists developing algorithms (Google, 2021). However, unlike the other two providers, Google is focusing less on hardware and more on quantum simulators and software to generalize quantum programming to develop novel quantum algorithms (Google, 2021). Google's open-source framework is called Cirq and is designed for developing and experimenting with NISQ algorithms on near-term quantum processors. The framework offers featured libraries and extensions such as OpenFermion to translate chemistry and materials problems into quantum circuits, TensorFlow Quantum for quantum machine learning, and qsim to simulate circuits integrated with Cirq to name a few (Google, 2021).

Microsoft invests in QC and offers several services, including the entire QC stack. The front end includes development tools that are included in the quantum development kit. The kit includes Q#, a high-level programming language for programming quantum algorithms, quantum simulators, libraries, and samples for arithmetic, chemistry, and for machine learning quantum development. Microsoft's quantum optimization solvers run on classical and accelerated computing resources in Azure (Nayak, 2021).

Amazon Web Services (AWS) offers a fully managed QC service to work with different types of QC and circuit simulators using a consistent set of development tools via Amazon Braket. Amazon does not develop or research its own quantum devices and provides hardware from other providers in the cloud (Amazon, 2021). The quantum devices made available are D-Wave's quantum annealers (Amazon b, 2021).

Applications of Quantum Computing in Supply Chain

Based on (Gil Dario et al., 2018), (Mohseni et al., 2017), applications of QC relevant to this review can be grouped into three main areas: optimization (e.g., travel and transportation, supply chain management, network infrastructure, traffic control, workload scheduling), artificial intelligence and machine learning (e.g., sampling, adaptive vendor and customer interactions, decision support, training, security), and logistics. Selected projects identified in the review illustrate the advances and applications developed.

Optimization. (D-Wave Systems Inc., 2021) demonstrates how Sigma-i is using D-Wave's quantum annealer to solve complex business relevant problems, here the scheduling problem in exceptional conditions of a pandemic. A recent topic due to COVID-19. (Ajagekar et al, 2020) proposes hybrid models and methods that effectively leverage the complementary strengths of deterministic algorithms and QC techniques to overcome the combinatorial complexity of solving large mixed-integer programming problems, such as the molecular conformation problem, job-shop scheduling problems, manufacturing cell formation problems, and vehicle routing problems. (Ajagekar and You, 2019) explores the applications of QC to energy systems optimization problems, comparing the gate-based model and annealers. Problems treated are facility location-allocation, unit commitment, and heat exchanger network synthesis. In (Chai et al., 2014), problems and applications to be found in graph theory, graph-based knowledge bases, and graph databases are mentioned.

Artificial Intelligence and Machine Learning. Sharma, 2020 investigates the integration of quantum circuits into traditional machine learning (ML) to perform classifications using breast cancer dataset. U. Alvarez-Rodriguez, M. Sanz et al., (2018) present the first experimental realization of a quantum artificial life algorithm in a QC. The quantum biomimetic protocol encodes tailored quantum behaviors belonging to living systems, namely, self-replication, mutation, the interaction between individuals, and death, into the QC of IBM. Dunjko and Briegel (2018) is a comprehensive review on artificial intelligence (AI), and machine learning (ML) in the quantum domain. This review describes the main ideas, recent developments, and progress in a broad spectrum of research investigating ML and AI combined with QC. The focus of the review is mainly on the utilization of quantum annealers and D-Wave Systems. Benedetti et al., (2015) is a case study of using quantum annealers to efficiently sample Boltzmann distributions, which can serve as applications for efficient deep-learning architectures.

Logistics. Harwood et al., (2021) solves the inventory and vehicle routing problems with time windows using QC, and comments on their strengths and weaknesses using a limited size of qubits on IBM's quantum gate computers. The authors have indicated hardware requirements of tens of thousands of logical qubits to solve real-world business problems. Clark et al., (2019) investigates the problem of routing multiple robots in real-time. They show that producing valid solutions for the problem of multi-robot routing is achievable with the current quantum annealing technology. D-Wave Systems Inc., (2019) optimizes the route of collecting waste for a smart city of the future using quantum annealing. As a result, the researcher was able to reduce the CO2 emissions by approximately 57% only by using the powerful QC for the computations.

RESEARCH DIRECTIONS

Research in QC and the (potential) applications of QC are growing as QC enables a leap in computation speed to solve even the most complex problems in supply chain management. IBM's vision (Gil Dario et al., 2018) of the future of QC and business-relevant use cases is in line with the view of many other companies, such as DHL (DHL, 2021), which predicts QC as a low-impact technology trend in five to ten years. IBM plans to reach the 1'000 qubits by the end of 2023. According to a market intelligence report (Rogers and Sanders, 2020), 4% of large companies surveyed in 2021 and 13% in 2024 are already experimenting with QC or plan to do so. 35% in 2021 and 68% in 2024 of respondents are at least thinking about QC - more than half of respondents said their interest in quantum computing was to gain a competitive advantage.

With the technology proliferating at a high acceleration rate, companies like IBM, D-Wave, Google, Microsoft, and Amazon are racing to develop QC and make it available as a service, and pave the way for other, highly specialized quantum computing services dedicated solving to specific problems. Moreover, QC has become readily accessible without the need for expert knowledge in quantum mechanics through frameworks such as Qiskit, Ocean, Cirq, Q#, Amazon Bracket and others. Predictions of several studies speak of the coming quantum era, and IBM's roadmap promises quantum hardware capable of tackling real-world applications by the end of 2023 (IBM, 2020).

Interestingly, the onset of the quantum area coincides with the area of supply chain hyperconnectivity, data-driven supply chain management, and the need to build resilient, and sustainable supply chains. From the combination arise several broad research streams:

- (1) Which types of problems in supply chain and logistics are better solved by which type of quantum computer and algorithm? This question relates to modeling and quantitative research and should include establishing comparisons over classical computing and algorithms and focus also on usability and acceptance among the expert users of supply chain software, as well as the composition and project methodologies in adoption.
- (2) How can supply chains transcend to the quantum area? This question concerns the strategy, process, and technology considerations in supply chain, as well as geographical and industry

considerations, e.g. the general maturity of supply chains, networks, and infrastructure and whether by using quantum computation, more intelligent solutions are enabled for geographies with specific constraints – be it a region with less financial resources or less space to place or depending on specific logistics infrastructure.

- (3) What type of competencies, skills and thus education is needed for supply chains and the communities they serve to reap the rewards of quantum computing, and how to teach those? This question also entails how supply chain staff can be re-skilled on a mass scale to use quantum as to circumnavigate the scarcity of supply chain tech talent we find with supply chain information technology today, and to prevent unwanted effects in algorithm design which are likely to potentiate with quantum (e.g. biases and discrimination).
- (4) On a meta-level, it is to be expected that the development of quantum computing and algorithms will inspire, in turn, the development of novel classical algorithms and computers. This quantum-classical competition may spark the development of even more supply chain computational options, and foster innovation in the domain. Fostering reciprocal scientific exchange and progress, and looking into the two fields simultaneously may be a key skill for current and future researchers in supply chain and logistics.

REFERENCES

- "A little bit, better," The Economist, Jun. 20, 2015. Accessed: Aug. 30, 2021. [Online]. Available: https://www.economist.com/science-and-technology/2015/06/20/a-little-bit-better
- "ETH Zürich und PSI gründen Quantum Computing Hub | Unsere Forschung | Paul Scherrer Institut (PSI)." https://www.psi.ch/de/media/forschung/eth-zurich-und-psi-grundenquantum-computing-hub (accessed Sep. 06, 2021).
- "List of QC simulators | Quantiki," Aug. 01, 2021. https://quantiki.org/wiki/list-qc-simulators (accessed Aug. 30, 2021).
- "Quantencomputer: ETH Zürich und PSI gründen Quantum Computing Hub." https://www.nzz.ch/wissenschaft/quantencomputer-eth-zuerich-und-psi-gruendenquantum-computing-hub-ld.1615124?reduced=true (accessed Sep. 06, 2021).
- A. Ajagekar and F. You, "Quantum computing for energy systems optimization: Challenges and opportunities," Energy, vol. 179, pp. 76–89, Jul. 2019, doi: 10.1016/J.ENERGY.2019.04.186.
- A. Ajagekar, T. Humble, and F. You, "Quantum computing based hybrid solution strategies for large-scale discrete-continuous optimization problems," Computers & Chemical Engineering, vol. 132, p. 106630, Jan. 2020, doi: 10.1016/J.COMPCHEMENG.2019.106630.
- A. Asfaw, K. Ferguson, and J. Weaver, "IBM offers quantum industry's first developer certification," 2021. https://www.ibm.com/blogs/research/2021/03/quantum-developer-certification/ (accessed Aug. 30, 2021).

- A. Cho, "The biggest flipping challenge in quantum computing," Science, Jul. 2020, doi: 10.1126/SCIENCE.ABD7332.
- A. Das and B. K. Chakrabarti, "Colloquium: Quantum annealing and analog quantum computation," Reviews of Modern Physics, vol. 80, no. 3, p. 1061, Sep. 2008, doi: 10.1103/RevModPhys.80.1061.
- Amazon, "Amazon Braket Quantum Computers Amazon Web Services," 2021. https://aws.amazon.com/braket/quantum-computers/ (accessed Sep. 22, 2021).
- Amazon, "Quantum Computing Service—Amazon Braket–Amazon Web Services," 2021. https://aws.amazon.com/braket/ (accessed Sep. 22, 2021).
- C. Baldassi and R. Zecchina, "Efficiency of quantum vs. classical annealing in nonconvex learning problems," Proceedings of the National Academy of Sciences, vol. 115, no. 7, pp. 1457– 1462, Feb. 2018, doi: 10.1073/PNAS.1711456115.
- C. Nay and IBM Research, "IBM Opens Quantum Computation Center in New York; Brings World's Largest Fleet of Quantum Computing Systems Online, Unveils New 53-Qubit Quantum System for Broad Use," Sep. 18, 2019. https://newsroom.ibm.com/2019-09-18-IBM-Opens-Quantum-Computation-Center-in-New-York-Brings-Worlds-Largest-Fleet-of-Quantum-Computing-Systems-Online-Unveils-New-53-Qubit-Quantum-System-for-Broad-Use (accessed Aug. 24, 2021).
- C. Nayak, "Full stack ahead: Pioneering quantum hardware allows for controlling up to thousands of qubits at cryogenic temperatures - Microsoft Research," Jan. 27, 2021. https://www.microsoft.com/en-us/research/blog/full-stack-ahead-pioneering-quantumhardware-allows-for-controlling-up-to-thousands-of-qubits-at-cryogenic-temperatures/ (accessed Sep. 19, 2021).
- D D-Wave Systems Inc., "What Is Leap?," 2021. https://support.dwavesys.com/hc/enus/articles/360005503634-What-Is-Leap- (accessed Sep. 27, 2021).
- D. Deutsch, "Quantum theory, the Church–Turing principle and the universal quantum computer," Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences, vol. 400, no. 1818, pp. 97–117, Jul. 1985, doi: 10.1098/RSPA.1985.0070.
- D. Leprince-Ringuet, "Quantum computing: Quantum annealing versus gate-based quantum computers," ZDNet, 2021. https://www.zdnet.com/article/quantum-computing-quantum-annealing-versus-gate-based-quantum-computers/ (accessed Aug. 14, 2021).
- D. Matthews, "How to get started in quantum computing," Nature, vol. 591, no. 7848, pp. 166– 167, Mar. 2021, doi: 10.1038/D41586-021-00533-X.
- DHL, "Logistics Trend Radar, Delivering Insight Today, Creating Value Tomorrow | DHL | Global." https://www.dhl.com/global-en/home/insights-and-innovation/insights/logistics-trendradar.html (accessed Sep. 01, 2021).
- D-Wave System Inc., "Quantum Computing," 2021. https://www.dwavesys.com/learn/quantumcomputing/ (accessed Aug. 30, 2021).

- D-Wave Systems Inc., "About D-Wave: The Practical Quantum Computing Company," 2021. https://www.dwavesys.com/company/about-d-wave/ (accessed Sep. 10, 2021).
- D-Wave Systems Inc., "Building the Smart Cities of the Future Image: example of waste collection for multiple buildings Groovenauts and Mitsubishi Estate: Creating Sustainable Cities through Waste Collection Optimization," 2019. Accessed: Aug. 31, 2021. [Online]. Available:

https://www.dwavesys.com/media/bq5kh520/dwave_groovenauts_case_study_v3.pdf

- D-Wave Systems Inc., "D-Wave Details Product Expansion & Cross Platform Roadmap," Oct. 05, 2021. https://www.dwavesys.com/company/newsroom/press-release/let-s-get-practical-d-wave-details-product-expansion-cross-platform-roadmap/ (accessed Sep. 26, 2021).
- D-Wave Systems Inc., "Education D-Wave Government," 2021. https://dwavefederal.com/education/ (accessed Sep. 27, 2021).
- D-Wave Systems Inc., "Quantum Computing for Developers," 2021. https://www.dwavesys.com/solutions-and-products/developer/ (accessed Sep. 27, 2021).
- D-Wave Systems Inc., "Sigma-i: Streamlining Personnel Management in the Time of COVID," 2021. Accessed: Aug. 31, 2021. [Online]. Available: www.dwavesys.com/d-wave-launch
- D-Wave Systems Inc., "Technical Description of the D-Wave Quantum Processing Unit," May 2021. Accessed: Aug. 25, 2021. [Online]. Available: https://docs.dwavesys.com/docs/latest/doc_qpu.html
- D-Wave Systems Inc., "The AdvantageTM Quantum Computer," 2021. https://www.dwavesys.com/solutions-and-products/systems/ (accessed Sep. 01, 2021).
- E. Farhi, J. Goldstone, S. Gutmann, and M. Sipser, "Quantum Computation by Adiabatic Evolution," Jan. 2000, Accessed: Aug. 27, 2021. [Online]. Available: https://arxiv.org/abs/quant-ph/0001106v1
- E. Gibney, "D-Wave upgrade: How scientists are using the world's most controversial quantum computer," Nature, vol. 541, no. 7638, pp. 447–448, Jan. 2017, doi: 10.1038/541447b.
- G. E. Santoro, R. Martoňák, E. Tosatti, and R. Car, "Theory of Quantum Annealing of an Ising Spin Glass," Science, vol. 295, no. 5564, pp. 2427–2430, Mar. 2002, doi: 10.1126/SCIENCE.1068774.
- G. G. Guerreschi and A. Y. Matsuura, "QAOA for Max-Cut requires hundreds of qubits for quantum speed-up," Scientific Reports 2019 9:1, vol. 9, no. 1, pp. 1–7, May 2019, doi: 10.1038/s41598-019-43176-9.
- Gil Dario et al., "Quantum computing is coming to your business," 2018. Accessed: Sep. 01, 2021. [Online]. Available: https://www.ibm.com/thought-leadership/institute-businessvalue/report/quantumstrategy

- Google, "Quantum Computer Datasheet," May 2021. Accessed: Sep. 07, 2021. [Online]. Available: https://quantumai.google/hardware/datasheet/weber.pdf
- Google, "Quantum Computing Service | Google Quantum AI," 2021. https://quantumai.google/quantum-computing-service (accessed Sep. 19, 2021).
- Google, "Research libraries and tools | Cirq | Google Quantum AI," 2021. https://quantumai.google/cirq/ecosystem (accessed Sep. 19, 2021).
- Google, "Software | Google Quantum AI," 2021. https://quantumai.google/software (accessed Sep. 19, 2021).
- H.-S. Zhong et al., "Quantum computational advantage using photons," Science, vol. 370, no. 6523, pp. 1460–1463, Dec. 2020, doi: 10.1126/SCIENCE.ABE8770.
- IBM Institute for Business Value, The Quantum Decade: A Playbook for Achieving Awareness, Readiness, and Advantage. IBM Institute for Business Value, 2021. [Online]. Available: https://www.ibm.com/thought-leadership/institute-business-value/report/quantumdecade
- IBM, "Become a Member of IBM Quantum Network," 2021. https://www.ibm.com/quantumcomputing/network/members/ (accessed Aug. 19, 2021).
- IBM, "ExxonMobil and World's Leading Research Labs Collaborate with IBM to Accelerate Joint Research in Quantum Computing," Jan. 09, 2019. https://newsroom.ibm.com/2019-01-08-ExxonMobil-and-Worlds-Leading-Research-Labs-Collaborate-with-IBM-to-Accelerate-Joint-Research-in-Quantum-Computing (accessed Oct. 07, 2021).
- IBM, "IBM Professional Certification Program IBM Certified Associate Developer Quantum Computation using Qiskit v0.2X," 2021. https://www.ibm.com/certify/cert?id=C0010300 (accessed Aug. 30, 2021).
- IBM, "IBM Unveils World's First Integrated Quantum Computing System for Commercial Use," Jan. 08, 2019. https://newsroom.ibm.com/2019-01-08-IBM-Unveils-Worlds-First-Integrated-Quantum-Computing-System-for-Commercial-Use (accessed Oct. 07, 2021).
- IBM, "IBM's Roadmap For Scaling Quantum Technology," 2020. https://www.ibm.com/blogs/research/2020/09/ibm-quantum-roadmap/ (accessed Aug. 24, 2021).
- IBM, "Quantum Computer Services IBM Quantum Network," 2021. https://www.ibm.com/quantum-computing/ibm-q-network/ (accessed Aug. 19, 2021).
- Intel, "Intel Debuts 2nd-Gen Horse Ridge Cryogenic Quantum Control Chip," Dec. 03, 2020. https://www.intel.com/content/www/us/en/newsroom/news/2nd-gen-horse-ridgecryogenic-quantum-control-chip.html (accessed Oct. 07, 2021).
- Intel, "The Future of Quantum Computing is Counted in Qubits," May 02, 2018. https://newsroom.intel.com/news/future-quantum-computing-countedqubits/#gs.absoxd (accessed Sep. 01, 2021).

- J. A. Smolin and G. Smith, "Classical signature of quantum annealing," Frontiers in Physics, vol. 2, p. 52, Sep. 2014, doi: 10.3389/fphy.2014.00052.
- J. Cai, W. G. Macready, and A. Roy, "A practical heuristic for finding graph minors," Jun. 2014, Accessed: Aug. 27, 2021. [Online]. Available: https://arxiv.org/abs/1406.2741v1
- J. Clark, T. West, J. Zammit, X. Guo, L. Mason, and D. Russell, "Towards Real Time Multi-robot Routing using Quantum Computing Technologies," 2019, doi: 10.1145/3293320.3293333.
- J. D. Hidary, Quantum Computing: An Applied Approach, 1st ed. 2019. Cham: Springer International Publishing, 2019.
- J. Kelly, "Google AI Blog: A Preview of Bristlecone, Google's New Quantum Processor," Google, Mar. 05, 2018. https://ai.googleblog.com/2018/03/a-preview-of-bristlecone-googlesnew.html (accessed Sep. 01, 2021).
- J. Martinis, "Google AI Blog: Quantum Supremacy Using a Programmable Superconducting Processor," 2019. https://ai.googleblog.com/2019/10/quantum-supremacy-usingprogrammable.html (accessed Oct. 07, 2021).
- J. Preskill, "Quantum Computing in the NISQ era and beyond," Quantum, vol. 2, Jan. 2018, doi: 10.22331/q-2018-08-06-79.
- K. Singh, K. L. Chuan, A. Ekert, C. C. Theng, J. Hogan, and E. Tan, "Two Decades of Quantum Information in Singapore," 50 Years of Science in Singapore, pp. 361–387, Feb. 2017, doi: 10.1142/9789813140905_0021.
- L. K. Grover, "A fast quantum mechanical algorithm for database search," Proceedings of the Annual ACM Symposium on Theory of Computing, vol. Part F129452, pp. 212–219, May 1996, Accessed: Aug. 13, 2021. [Online]. Available: https://arxiv.org/abs/quantph/9605043v3
- L. Petit et al., "Universal quantum logic in hot silicon qubits," Nature 2020 580:7803, vol. 580, no. 7803, pp. 355–359, Apr. 2020, doi: 10.1038/s41586-020-2170-7.
- Laura Hennemann, "«Ziel ist ein experimenteller Quantencomputer im Kanton Aargau» | Unsere Forschung | Paul Scherrer Institut (PSI)," May 03, 2021. https://www.psi.ch/de/media/forschung/ziel-ist-ein-experimenteller-quantencomputerim-kanton-aargau (accessed Sep. 06, 2021).
- M. Benedetti, J. Realpe-Gómez, R. Biswas, and A. Perdomo-Ortiz, "Estimation of effective temperatures in quantum annealers for sampling applications: A case study with possible applications in deep learning," Physical Review A, vol. 94, no. 2, Oct. 2015, doi: 10.1103/physreva.94.022308.
- M. F. Riedel, D. Binosi, R. Thew, and T. Calarco, "The European quantum technologies flagship programme," Quantum Science and Technology, vol. 2, no. 3, p. 030501, Jun. 2017, doi: 10.1088/2058-9565/AA6ACA.
- M. G. Raymer and C. Monroe, "The US National Quantum Initiative," Quantum Science and Technology, vol. 4, no. 2, p. 020504, Feb. 2019, doi: 10.1088/2058-9565/AB0441.

- M. Grimm, A. Beckert, G. Aeppli, and M. Müller, "Universal Quantum Computing Using Electronuclear Wavefunctions of Rare-Earth Ions," PRX Quantum, vol. 2, no. 1, Jan. 2021, doi: 10.1103/PRXQUANTUM.2.010312.
- M. Mohseni et al., "Commercialize quantum technologies in five years," Nature, vol. 543, no. 7644, pp. 171–174, Mar. 2017, doi: 10.1038/543171a.
- M. Murgia and R. Waters, "Google claims to have reached quantum supremacy," Financial Times, Sep. 20, 2019. https://www.ft.com/content/b9bb4e54-dbc1-11e9-8f9b-77216ebe1f17 (accessed Aug. 26, 2021).
- M. W. Johnson et al., "Quantum annealing with manufactured spins," Nature 2011 473:7346, vol. 473, no. 7346, pp. 194–198, May 2011, doi: 10.1038/nature10012.
- N. Datta, "Quantum Entropy and Quantum Information," Les Houches Summer School Proceedings, vol. 83, no. C, pp. 395–466, Jan. 2006, doi: 10.1016/S0924-8099(06)80046-6.
- New blueprint for more stable quantum computers | Our Research | Paul Scherrer Institut (PSI)." https://www.psi.ch/en/media/our-research/new-blueprint-for-more-stable-quantumcomputers (accessed Sep. 06, 2021).
- P. Ball, "Physicists in China challenge Google's 'quantum advantage," Nature, vol. 588, no. 7838, p. 380, Dec. 2020, doi: 10.1038/D41586-020-03434-7.
- P. Benioff, "The computer as a physical system: A microscopic quantum mechanical Hamiltonian model of computers as represented by Turing machines," Journal of Statistical Physics 1980 22:5, vol. 22, no. 5, pp. 563–591, May 1980, doi: 10.1007/BF01011339.
- P. Knight and I. Walmsley, "UK national quantum technology programme," Quantum Science and Technology, vol. 4, no. 4, p. 040502, Oct. 2019, doi: 10.1088/2058-9565/AB4346.
- P. M. Pardalos, "Constrained Global Optimization: Algorithms and Applications." Springer Berlin Heidelberg, Berlin, Heidelberg, 1987.
- P. W. Shor, "Algorithms for quantum computation: discrete logarithms and factoring," pp. 124– 134, Dec. 2002, doi: 10.1109/SFCS.1994.365700.
- R. Martoňák, G. E. Santoro, and E. Tosatti, "Quantum annealing of the traveling-salesman problem," Physical Review E, vol. 70, no. 5, p. 057701, Nov. 2004, doi: 10.1103/PhysRevE.70.057701.
- R. P. Feynman, "Simulating physics with computers," International Journal of Theoretical Physics 1982 21:6, vol. 21, no. 6, pp. 467–488, Jun. 1982, doi: 10.1007/BF02650179.
- S. Harwood, C. Gambella, D. Trenev, A. Simonetto, D. Bernal Neira, and D. Greenberg, "Formulating and Solving Routing Problems on Quantum Computers," IEEE Transactions on Quantum Engineering, vol. 2, pp. 1–17, Jan. 2021, doi: 10.1109/TQE.2021.3049230.
- S. Jordan and et. al., "Quantum Algorithm Zoo." https://quantumalgorithmzoo.org/ (accessed Aug. 30, 2021).

- S. Sharma, "QEML (Quantum Enhanced Machine Learning): Using Quantum Computing to Enhance ML Classifiers and Feature Spaces," Feb. 2020, Accessed: Aug. 27, 2021. [Online]. Available: https://arxiv.org/abs/2002.10453v3
- T. Ray, "What the Google vs. IBM debate over quantum supremacy means," ZDNet, Oct. 30, 2019. https://www.zdnet.com/google-amp/article/what-the-google-v-ibm-debate-overquantum-means/ (accessed Aug. 12, 2021).
- U. Alvarez-Rodriguez, M. Sanz, L. Lamata, and E. Solano, "Quantum Artificial Life in an IBM Quantum Computer," Scientific Reports 2018 8:1, vol. 8, no. 1, pp. 1–9, Oct. 2018, doi: 10.1038/s41598-018-33125-3.
- V. Dunjko and H. J. Briegel, "Machine learning & artificial intelligence in the quantum domain: a review of recent progress," Reports on Progress in Physics, vol. 81, no. 7, p. 074001, Jun. 2018, doi: 10.1088/1361-6633/AAB406.
- V. N. Smelyanskiy et al., "A Near-Term Quantum Computing Approach for Hard Computational Problems in Space Exploration," Apr. 2012, Accessed: Aug. 27, 2021. [Online]. Available: https://arxiv.org/abs/1204.2821v2
- O. Rogers and J. Sanders, "Enterprises Are Preparing to Quantum Leap Today," Sep. 2020. Accessed: Sep. 12, 2021. [Online]. Available: https://www.dwavesys.com/see-whatspossible/reinvent-our-world/